

Eminent Domain versus Government Purchase of Land Given Imperfect Information About Owners' Valuations

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Governments employ two basic policies for acquiring land: taking it through exercise of their power of eminent domain; and purchasing it. The social desirability of these two policies is compared in a model in which the government's information about landowners' valuations is imperfect. Under this assumption, the policy of purchase possesses the market test advantage that the government obtains land only if an owner's valuation is low enough that he is willing to sell it. However, the policy suffers from a drawback when the land that the government needs is owned by many parties. In that case, the government's acquisition will fail if any of the owners refuses to sell. Hence, the policy of eminent domain becomes appealing if the number of owners of the land is large. This conclusion holds whether the land that the government seeks is a specific parcel or instead may be located anywhere in a region.

1. Introduction

Governments generally enjoy the ancient right of eminent domain, which is to say, the right to take land by fiat, usually on the condition that they pay fair compensation to the landowner.¹ But governments may, of course, also obtain land by purchasing it, raising the question why governments need the right of eminent domain.

In this article, I consider an aspect of that question by comparing the social desirability of eminent domain to government purchase of land in a simple model of land

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¹ On the right of eminent domain and the requirement to pay compensation in the legal system of the United States, see, for example, Dukeminier et al. (2006), ch. 12. On the history of eminent domain or its equivalents in the United States and other countries, see, for instance, Nichols (2006), sec. 1-1.

acquisition. The key feature of the model is that the government's knowledge of landowners' valuations is imperfect. Under this assumption, the policy of purchase possesses the classic advantage that acquisition of land is subjected to the *market test* – the government obtains a parcel of land only if its value to the owner is low enough that the owner is willing to sell to the government. In contrast, eminent domain involves no market test, implying that the government might turn out to take a parcel of land even though its value to the owner exceeds its social value. The policy of purchase suffers from a difficulty, however, when the land that the government requires is held by multiple owners, for then the government needs all the owners to sell in order to proceed. The necessity of such *unanimity* in owners' decisions to sell constitutes an acute disadvantage of the policy of purchase when the number of owners grows large, for in that context the likelihood that some owner will reject the government's price offer becomes high.

Consequently, the policy of eminent domain tends to be appealing when the number of owners is large, whereas the policy of purchase tends to be better when the number of owners is low. This is the major conclusion of the present article, but as its summary below will make clear, the conclusion is qualified and amplified in important ways, especially in relation to whether the land the government requires is a specific parcel, only a contiguous parcel that may be located anywhere within a region, or a parcel the character of which is governed only by the marginal social value of land.

In Section 2 I state the main assumptions of the model. I suppose that the government places a social value on the land it needs and that the values that private owners place on the land are not observable to the government but are drawn from a

known distribution. Under eminent domain, the government may take land from private owners but must pay appropriate compensation for doing so,² where such compensation is interpreted as the expected private value of land. Under the policy of purchase, the government makes a single price offer for the land. If the government obtains the land, it realizes the social value of the land, but the payment that the government makes (in compensation for a taking or in the purchase price) is assumed to involve a social cost of funds (because of the cost of raising funds). The government acts to maximize social welfare.

In Section 3 I address the case where a single owner holds a specific parcel of land that the government seeks. Under the policy of eminent domain, the government will take the land if its social value minus the social cost of paying just compensation exceeds its expected private value.³ Under the policy of purchase, the government's social welfare-maximizing price offer must be positive but less than the social value of the parcel, owing to the social cost of funds.⁴ Purchase is often socially superior to eminent domain, the reason being the market test advantage of that policy, that when the owner's value turns out to be high and government acquisition would not be socially desirable, the owner will reject the price offer. Still, it is possible that eminent domain

² I make the assumption that compensation must be paid because this is a legal requirement (see the previous note). It will be clear, however, that in the model that I examine, the policy of eminent domain without compensation would be superior to eminent domain with compensation, because the social cost associated with compensation (to be explained) would be avoided. Hence, the main conclusion that I reach concerning the superiority of eminent domain to purchase when the number of landowners is large would certainly hold for eminent domain without compensation.

³ Suppose that the government seeks an acre parcel with a social value of \$1 million, that the social cost of a dollar of funds is \$.10, and that the expected private value of the acre is \$100,000. Then the government will take the acre, since its social value net of the social cost of paying \$100,000 in fair compensation would be \$990,000, which exceeds its expected private value of \$100,000.

⁴ Thus, in the example of the previous footnote, the optimal price offer must be positive but below the \$1 million social value of the land.

would be superior to purchase because of a potential *acquisition cost advantage* enjoyed under the former policy. Namely, under eminent domain, the government acquires land at a fair compensation cost equal to its expected value to owners, whereas under the policy of purchase the government may have to (and thus will find it desirable to) offer substantially more than the expected value of the land in order to acquire the land with high probability.⁵

In Section 4 I examine the case where multiple owners hold parcels making up the parcel that the government wishes to obtain. In particular, I assume that there are n owners and that their values per acre are independently and identically distributed. Under eminent domain, it is optimal for the government to take the parcel (meaning take all n owners' parcels) under the same condition as in the single owner case. Under the policy of purchase, the government's optimal price offer is positive but is not necessarily less than the social value of the land. The explanation of this somewhat counterintuitive observation that the optimal price offer might exceed the social value per acre is that such a high offer may be useful in inducing every single owner to sell, as that is necessary for the government to acquire its parcel and for the social value of the parcel to be realized.⁶

⁵ In the example of footnote 3, suppose that there are two possible owners' values: \$50,000, with probability 50%; and \$150,000, with probability 50%. Under eminent domain, the government would pay fair compensation of \$100,000. Under purchase, it can be verified that the government would choose the price offer of \$150,000 in order to obtain the land for sure. Thus, the acquisition cost – and therefore the social cost of funds – would be higher under the policy of purchase. Hence, social welfare would be lower under the policy of purchase. In this example, there is no market test advantage of the policy of purchase because the distribution of values is discrete. In the model, however, there always is such an advantage because the distribution of owners' values is continuous and unbounded, but the possible advantage of eminent domain still exists.

⁶ In the example of footnote 3, the optimal offer per acre could exceed \$1 million, for instance, it could be \$2 million per acre. If there are many owners of the parcel that the government seeks, it could be likely that one among them would place a value on land of \$2 million per acre, and to induce him to sell, the offer per acre would have to be at least \$2 million. Such a high offer might not lead to very much inefficiency because most of the owners who would accept the offer might be likely to place values on land that are below its social value of \$1 million per acre. The optimality of offers exceeding the social value of

With regard to the comparison of eminent domain versus purchase in the multiple owner case, either policy could be superior if the number of owners is not large, but eminent domain must be superior to purchase if the number of owners is sufficiently large, assuming that eminent domain leads to positive social welfare. The kernel of the argument for this conclusion (the central one of the article) is, as was mentioned, that the problem of making a price offer that all owners will accept becomes great as n grows. In particular, I show that the probability of the government's making a successful purchase tends to zero with n (even though the price is optimally adjusted as n increases). Moreover, the probability that government acquisition of the parcel would in fact be socially desirable under eminent domain tends to one with n . Thus, when the number of owners is large, the sense in which the policy of eminent domain is superior to the policy of purchase is strong.

In Section 5 I consider two extensions of the model. In the first, the government does not require a specific parcel of land but rather any parcel in a relevant region that is of the needed size. The main point demonstrated is that the conclusion that the policy of eminent domain is superior to purchase when the number of owners is sufficiently large continues to hold. One might think otherwise because, when the government can obtain the land it needs anywhere within a region, it does require unanimous approval of a price offer from a named group of owners. But if, as is assumed in this extension, the land that the government seeks is contiguous, the government does require that some set of

land is not anomalous, for I show that offers of this type must be optimal as n grows large under broad conditions.

neighboring landowners all give their approval to a price offer.⁷ This turns out to be a strong enough hurdle that the policy of purchase will fail with high likelihood if the number of owners is sufficiently large. In consequence, the comparison of purchase and of eminent domain is qualitatively the same as in Section 4, in which the government needs a fixed parcel of land with multiple owners.

In the second extension, the government places a social value on land but does not require a particular kind of parcel; the government merely places a constant social value per acre on land. In this case, eminent domain does not become attractive as the number becomes large, essentially because the social value of land to the government is only proportional to its amount.

In the conclusion, I comment on two final issues: the bearing of the analysis on the use of eminent domain on behalf of private developers; and the applicability of the analysis to property other than land.

Before beginning, let me comment on the relationship between this article and previous writing on eminent domain. The main contribution made here is the development of the notion that variability in individual valuation of land may lead to defeat of the policy of purchase and may justify eminent domain when the number of landowners is large. To my knowledge, this idea has not been analyzed in prior writing. However, legal literature on eminent domain and some economic articles informally advance the view that bargaining difficulties may justify eminent domain when the number of owners is large, referring to the problem that a subset of owners may attach high value to their land (the problem studied here) and also to the different problem of

⁷ I show that the problem of obtaining some group of neighboring landowners all to give their approval is equivalent to that of obtaining a type of run of successes in repeated Bernoulli trials.

“strategic holdout,” namely, that owners (especially the last few) may negotiate for a high price in order to extract some of the social surplus that would be gained by the government if their land was sold.⁸ Models of strategic holdout are studied in Strange (1995) and Miceli and Segerson (2007). Two other articles of note on eminent domain are Blume, Rubinfeld, and Shapiro (1984) and Hermalin (1995). They address primarily the effect of the government’s payment of compensation when it takes land on the private investment incentives of a single landowner; they are not concerned with comparing eminent domain to a policy of purchase, and thus not with the bargaining problems that arise under purchase when there are multiple landowners. See also Munch (1976), a largely empirical investigation of eminent domain.

2. Assumptions and Framework of Analysis

The government places a social value on a specific parcel of land that it needs to acquire, for instance, to build an airport at a particular location. For concreteness I call the parcel an acre. Let

s = social value of the acre of land needed by the government; $s > 0$.

If the government does not acquire the full acre of land, it will obtain no value. This assumption is made to reflect situations in which it is important for the government to possess at least a threshold quantity of land (airplanes require at least a minimum amount of runway for takeoff and landing).⁹ (Situations that do not have this character, in which

⁸ See, for example, Calabresi and Melamed (1972), pp. 1106-1108, and Munch (1976), pp 476-479.

⁹ If I were instead to assume that the social value of land is rising and convex in the quantity of land, the qualitative results would be similar to those I find.

the social value of land is proportional to the quantity of land, rather than depending on a threshold quantity, are discussed in the second extension of Section 5.)

Land is owned at the outset by private parties, called owners. An owner's value per acre of land is drawn from a distribution of possible values. Let

v = value per acre of land to an owner,

where

$f(v)$ = probability density of v , $f(v) > 0$ on $[0, \infty]$ and 0 elsewhere; $F(v)$ is the distribution function of $f(v)$.

The value v is known to an owner but not observed by the government; the government knows f . Let

n = number of owners.

In the case of $n > 1$, multiple owners (each of whom owns a different parcel), the value per acre of the i^{th} owner is denoted v_i , where the v_i are independently drawn from the distribution with density f .

Under the policy of *eminent domain*, the government may take land, and if so must pay to the owner(s) fair compensation, assumed to be the expected value of the land per acre, namely, $E(v)$, where E stands for expectation.

Under the policy of *purchase*, the government makes a single price offer to owners. Let

x = price offer of the government per acre.

An owner will thus accept the offer if and only if $v_i < x$.¹⁰ That there is a single offer applying to all n owners is due to the assumption that the government cannot observe the v_i .¹¹ I further describe the policy of purchase below.

The government bears a positive social cost per dollar of funds, which may be interpreted as due to the administrative expense of taxation or to its distortionary effects.

Let

c = social cost per dollar of government funds; $c > 0$.

Social welfare W equals the value of land (to whomever enjoys it) minus the social cost of government funds expended. The government acts to maximize social welfare.

3. A Single Party Owns The Land Sought By The Government

Assume here that the acre needed by the government has one owner. This case is worth studying because important aspects of the comparison of eminent domain and purchase are best understood in isolation from issues concerning multiple owners.

Note that it is first-best for the government to obtain the land from the owner if and only if $s > v$.

Under eminent domain, if the government takes the land, social welfare will be

$$(1) \quad s - cE(v),$$

because the government will obtain the benefit s from the land and incur the social cost $cE(v)$ from its payment of compensation of $E(v)$ for the taking. If the government does not take the land, social welfare will be

¹⁰ If $v_i = x$, the owner will be indifferent about accepting the offer, and I adopt the convention that he will not accept it. I make similar assumptions below without further comment.

¹¹ I remark on the outcome if the government could observe the v_i at the end of Section 4.

$$(2) \quad E(v)$$

since the owner will make use of the land.

Under the policy of purchase, if the price offer is x , social welfare will be

$$(3) \quad W(x) = F(x)(s - cx) + \int_x^{\infty} v f(v) dv$$

because x will be accepted when v is below x and rejected when v is at least x . The government's problem is to maximize $W(x)$ over $x \geq 0$. Let x^* denote the optimal offer.¹²

The conclusions reached about the two policies are given below.

Proposition 1. (a) Under the policy of eminent domain, land is taken if and only if its social value s is sufficiently high, if and only if $s > (1 + c)E(v)$.

(b) Under the policy of purchase, the optimal offer x^* is positive, so that land is purchased with positive probability, and x^* is less than the social value of land s . In particular, x^* satisfies $x = [s - cF(x)/f(x)]/(1 + c)$.

(c) The social welfare comparison of the policies of eminent domain and of purchase is as follows:

(i) If the social value of land s does not exceed $(1 + c)E(v)$, so that under eminent domain there would not be a taking, then purchase is superior to eminent domain.

(ii) Otherwise, either policy could be superior; and in particular, purchase is superior to eminent domain for all c sufficiently low, and eminent domain is superior to purchase for all s sufficiently high. ■

Let me now discuss these conclusions. The proofs of conclusions that are not shown below are given in the Appendix.

¹² The optimal offer might not be unique, but for expositional ease I will describe it as if it were.

With regard to eminent domain and part (a), it follows from (1) and (2) that under eminent domain, the government will take land if and only if

$$(4) \quad s > (1 + c)E(v),$$

which is to say, when its social value exceeds the expected private value multiplied by the factor $(1 + c)$, reflecting the social cost of funds.

With regard to purchase and part (b), it can be shown that the problem of maximizing (3) over offers x has a solution x^* . The derivative of (3) with respect to x is

$$(5) \quad W'(x) = [s - (1 + c)x]f(x) - cF(x),$$

which is positive at $x = 0$. Hence, x^* must be positive. The explanation is that, regardless of s , there will be some chance that v is less than s , in which case government acquisition of land would be desirable if the social cost of funds c were not positive. But no matter how high c is, if the offer x is sufficiently low, the social cost of funds will be outweighed by the social value of the land. Since x^* is positive, we know that the optimal x^* is determined by the first-order condition

$$(6) \quad [s - (1 + c)x]f(x) - cF(x) = 0,$$

so that x^* must satisfy

$$(7) \quad x = [s - cF(x)/f(x)]/(1 + c) < s.$$

The intuition explaining that $x^* < s$ is as follows. If $x = s$ the allocation of land would be ideal, so that there would be no first-order loss in social welfare from lowering x (owners just induced to refuse the offer would obtain values only slightly below s). But there would be a first-order social savings achieved from lowering x because c is positive and the government spends less when its bid is accepted. Note too that, if c is zero, (7) implies that $x^* = s$ and the first-best outcome would result.

To compare social welfare under the two policies, assume first that s is low enough that (4) does not hold, meaning that under eminent domain there will be no taking. Then, under eminent domain social welfare will be $E(v)$. Under the policy of purchase, however, social welfare will be higher. To show this, observe that $W(0) = E(v)$, for if an offer of 0 is made, no owner will accept it. But I noted above that $W'(0) > 0$. Hence social welfare must exceed $E(v)$ at x^* . The explanation is that, under the policy of purchase, there is a positive probability that the owner's value will be lower than x^* and land will be acquired by the government; in other words, the market test advantage sometimes results in a beneficial acquisition under the policy of purchase.

Now assume that (4) holds, so that under eminent domain there will be a taking and social welfare will be $s - cE(v) > E(v)$. Let me first show that for all low c , purchase is superior to eminent domain; the underlying reason is that for low c the market test advantage, which allows individuals with high v to retain their land, is more important than the social cost of paying for land. In particular, if c is 0, social welfare under purchase exceeds social welfare under eminent domain: under purchase x^* is s , so purchase is made if and only if $s > v$ and the first-best outcome is achieved; under eminent domain, land is taken; hence, social welfare is higher under purchase by the amount

$$(8) \quad \int_s^{\infty} v f(v) dv > 0.$$

It follows that social welfare must be higher under purchase than under eminent domain for all c sufficiently low, since social welfare under each policy is continuous in c .

Let me now explain why eminent domain might be superior to purchase. As mentioned in the Introduction, the answer concerns a potential cost advantage to the

government. Specifically, when the government takes land under eminent domain, it pays $E(v)$, but when the government purchases land at its optimal offer price x^* , this offer may well exceed $E(v)$, and significantly so, because such an offer might be needed to obtain land with a high likelihood given the dispersion of the distribution of owners' values v . To illustrate, consider for simplicity a discrete distribution of v (like that in footnote 5): v is 50 with probability .5 and 150 with probability .5, so $E(v) = 100$; the social value s is 1,000, and c is .1. Under eminent domain, taking land and paying compensation of $E(v) = 100$ is optimal and social welfare is $s - cE(v) = 1,000 - .1(100) = 990$. Under the policy of purchase, the government's optimal offer is 150, because the government will wish to obtain the land with certainty rather than only with probability .5. Hence, social welfare is $1,000 - .1(150) = 985$. Social welfare is lower because the government has to pay (and wants to pay) more, 150, in order to obtain the land for sure. The acquisition cost advantage of eminent domain in a case like this constitutes a social welfare advantage because the cost c of funds is positive.

The intuition behind the foregoing discrete example suggests why it is true that if s is sufficiently high, eminent domain will be superior to purchase. If s is high, it will be socially desirable for the offer price x^* to be high, and higher than $E(v)$, implying that there will be a cost advantage obtained under eminent domain. It is true that the market test advantage will be lost under eminent domain. But as s grows, the market test advantage becomes insignificant in importance, because the probability that an owner's value v would exceed s tends toward 0.

4. Multiple Parties Own The Land Sought By The Government

Now assume that there are n individual owners of parcels making up the acre of land needed by the government. The owners' values per acre are v_1, \dots, v_n , which, recall, are independent draws from the distribution F , a random sample from F . For simplicity, I suppose that each owner's parcel is $1/n$ of the acre.¹³

Note that it is first-best for the government to acquire the acre if and only if $s > v_1/n + \dots + v_n/n = (v_1 + \dots + v_n)/n$, that is, if and only if s exceeds the sample mean.

Under eminent domain, I assume that if the government exercises its right to appropriate land, it takes the entire acre and pays each of the n owners $(1/n)E(v)$, as this is the expected value of each parcel.¹⁴ Hence, if the government takes land, its total payment to the n owners is $E(v)$. Accordingly, if the government takes the acre, social welfare is $s - cE(v)$, which is (1). If the government does not take the land, social welfare is $E(v_1/n + \dots + v_n/n) = E(v)$, which is (2). Thus, social welfare when there is a taking and when there is not is the same as in the single owner case.

Under the policy of purchase, I assume that the government's offer x holds only if all owners accept it, that is, if any owner rejects x , the government pays no one and all owners retain their land.¹⁵ The probability that the offer of x per acre is accepted by all owners is $F(x)^n$. In this event each owner receives $(1/n)x$, so that the total payment of the government is x . Hence, if the offer x is accepted by all owners, social welfare is $s - cx$.

¹³ The results to be obtained do not depend on this assumption, but the notation would become more burdensome were individual owners' parcel sizes to vary.

¹⁴ I do not consider the exercise of eminent domain in order to take less than the entire acre because the assumption is that the social value of less than the entire acre is 0.

¹⁵ If the government were to pay owners who accepted its bid when not all owners accepted, the analysis would not be qualitatively different, because, as will be seen, the key point of this section is that the government needs all owners to accept to obtain the social value from the land.

If x is not accepted by all owners, social welfare is $v_1/n + \dots + v_n/n$. The integral of social welfare when x is not accepted by all owners can be expressed as

$$(9) \int_0^{\infty} \dots \int_0^{\infty} [v_1/n + \dots + v_n/n] f(v_1) \dots f(v_n) dv_1 \dots dv_n \\ - \int_0^x \dots \int_0^x [v_1/n + \dots + v_n/n] f(v_1) \dots f(v_n) dv_1 \dots dv_n.$$

To explain, the first term is the sum of the n owners' values of their $1/n$ acre parcels under all possible outcomes of the v_i , and the second term is the sum of the owners' values when all owners accept x , so the difference must be the sum of owners' values when not all of them accept x . The first term in (9) equals $E(v)$, and the second term reduces to n similar terms, the first of which is

$$(10) \quad (1/n) \int_0^x \dots \int_0^x v_1 f(v_1) \dots f(v_n) dv_1 \dots dv_n \\ = (1/n) \int_0^x v_1 f(v_1) dv_1 \left(\int_0^x f(v_2) dv_2 \right) \dots \left(\int_0^x f(v_n) dv_n \right) = (1/n) F(x)^{n-1} \int_0^x v_1 f(v_1) dv_1.$$

There are n such terms, adding to

$$(11) \quad F(x)^{n-1} \int_0^x v f(v) dv.$$

Hence, social welfare under purchase given x equals

$$(12) \quad W_n(x) = F(x)^n (s - cx) + [E(v) - F(x)^{n-1} \int_0^x v f(v) dv].$$

The government's problem is to maximize $W_n(x)$ over $x \geq 0$.

The results in this n owner case are as follows, where $x^*(n)$ denotes the optimal offer given n .

Proposition 2. (a) Under the policy of eminent domain, land is taken if and only if its social value s is sufficiently high, if and only if $s > (1 + c)E(v)$.

(b) Under the policy of purchase,

(i) the optimal offer $x^*(n)$ is positive, so that land is purchased with positive probability, and $x^*(n)$ may exceed the social value of land s . In particular, $x^*(n)$ satisfies (13).

(ii) As n grows, the optimal offer $x^*(n)$ tends to a limit x^{**} determined by (15), where $x^{**} > x^*(1)$ and where $x^{**} > s$ for all c sufficiently low. Also, as n grows, the probability of successful purchase $F(x^*(n))^n$ tends to 0.

(c) The social welfare comparison of the policies of eminent domain and purchase is as follows:

(i) If the social value of land s does not exceed $(1 + c)E(v)$, so that under eminent domain there would not be a taking, then purchase is superior to eminent domain.

(ii) If s exceeds $(1 + c)E(v)$, then either policy could be superior; and in particular, purchase is superior to eminent domain for all c sufficiently low, and eminent domain is superior for all s sufficiently high.

(iii) If s exceeds $(1 + c)E(v)$, then for all n sufficiently high, eminent domain is superior to purchase. Indeed, as n grows, the probability that eminent domain is socially desirable tends to 1, whereas the probability that purchase would be successful tends to 0. ■

Let me explain and comment on these conclusions.¹⁶

¹⁶ Again, all claims not shown are demonstrated in the Appendix.

Part (a) is clear, for social welfare if there is taking and if not is as previously discussed in Section 3.

With regard to part (b), it can be shown that the problem of maximizing $W_n(x)$ has a solution x^* .

That x^* must be positive is essentially for the reason given in respect to Proposition 1. Namely, regardless of s , there will be some chance that the mean of the v_i is less than s , in which case government acquisition of land would be first-best; and no matter how high c is, if the offer x is sufficiently low, the social cost of funds will be of second-order significance.

Why, however, might x^* *exceed* the social value s of land, that is, why might it be desirable for government to offer more than the land is worth? In the case of a single owner, it was not desirable for the government's offer x to exceed s , for such an offer could only result in a welfare-lowering purchase from an owner with a $v > s$. In the case here of multiple owners, however, an $x > s$ may be needed to induce a high v_i owner to accept, along with *others* whose v_i are low and for that reason make government acquisition of the entire acre of land socially worthwhile.¹⁷

Consider the following discrete example with two owners:¹⁸ v is 1 with probability .9 and 11 with probability .1, s is 10, and c is .01. There are thus three pairs of possible values of v_1 and v_2 : (1, 1), with probability .81; (1, 11), with probability .18; and (11, 11), with probability .01. Observe that it is first-best for the government to

¹⁷ A closely related point is that in the multiple owner case, confronting an owner with a price s does not create a socially desirable market test, because that owner's decision to sell does not determine whether the land the government seeks is sold; the decision of all others as well matters.

¹⁸ It will be clear that a similar continuous example would also result in an $x^* > s$.

acquire the acre when the two owners' values are (1, 1) and (1, 11), for then the average values per acre are 1 and 6, respectively, below the social value of 10 for the acre; and it is first-best for the owners to retain their land when their values are (11, 11), for then their average value per acre is 11, exceeding the social value. Now compare the two possible offers x of 1 and 11.¹⁹ If $x = 1$, both owners will accept and the government will purchase the land only when the pair is (1, 1); social welfare will be $.81(10 - 1 - .01) + .18(6) + .01(11) = 8.47$. If $x = 11$, both owners will always accept; social welfare will be $10 - .01(11) = 9.89$, which is higher. The reason that social welfare rises when the offer is 11 – which exceeds the social value of 10 – is that an offer of 11 is necessary for a successful purchase when the pair is (1, 11), in order to attract the owner whose value per acre is 11. Further, this purchase is socially desirable because the other owner's value is low (making the average value 6, less than the social value of 10). It is true that an offer of 11 also results in a socially undesirable purchase when the owners' values are (11, 11), but this is a relatively unlikely event (its probability is .01, as opposed to the probability of .18 of (1, 11)). The intuition of this example underlies the result in part (iii) that, for all n sufficiently large, the optimal offer $x^*(n)$ must exceed s if c is not too high. That is, provided that c is not too high, when there are many buyers, it is always desirable to raise the offer somewhat above s in order to attract some high value owners, because they will be likely to be accompanied by other low value owners.

The first-order condition determining x^* , from differentiating $W_n(x)$, is

$$(13) \quad F(x)^{n-1}f(x)[n(s - cx) - x] - cF(x)^n - (n - 1)F(x)^{n-2}f(x) \int_0^x v f(v) dv = 0.$$

¹⁹ It is obvious that an offer higher than 1 and less than 11 cannot be optimal, as it would be accepted in the same circumstances as an offer of 1 but cost the government more.

The first term reflects the marginal expected benefit from raising x when bids are accepted. The second term is the marginal social cost due to the government's having to pay inframarginal individuals a higher amount. The third term measures the expected loss from the fact that when the marginal person is attracted to make a purchase, he brings with him $n - 1$ other owners who no longer derive values from their land.

Equation (13) can be rewritten as

$$(14) \quad (s - cx) = cF(x)/(nf(x)) + x/n + [(n-1)/(nF(x))] \int_0^x vf(v)dv.$$

As n grows large, (14) tends to

$$(15) \quad (s - cx) = F(x) \int_0^x vf(v)dv.$$

This can be shown to have a unique solution x^{**} to which $x^*(n)$ tends as n grows.

That $x^{**} > x^*(1)$ is explained by the fact that as n grows, it becomes more difficult to attract all the owners to accept, because some owner is likely to have a high value v_i .²⁰ Also, the closely-related intuition that I gave for why an offer exceeding s is possible would seem to be more important the higher n is; this helps to explain the result that $x^{**} > s$ for all positive c sufficiently low.

That the probability of a successful purchase $F(x^*(n))^n$ tends to 0 is due to the fact that, as n grows large, the odds of some person placing a high value on his property that exceeds the government's bid rise. Of course, the optimal bid $x^*(n)$ is chosen taking into account the presence of n owners, but the problem of obtaining acceptance from all n

²⁰ This intuition also suggests that $x^*(n)$ is increasing in n , but I have not been able to establish this.

remains. The proof follows from the point that $x^*(n)$ tends to a finite limit x^{**} , implying that $\lim_{n \rightarrow \infty} F(x^*(n))^n = \lim_{n \rightarrow \infty} F(x^{**})^n = 0$.

Now let us turn to the social welfare comparison of eminent domain and purchase. Claims (i) and (ii) are explained essentially as in the single owner case. For given n , purchase must be superior to eminent domain if under eminent domain, there would not be a taking – which is when $s < (1 + c)E(v)$ – for under purchase there is always some positive probability of purchase and of raising social welfare above the status quo, due to the market test advantage (however attenuated this is, due to there being n owners). And if under eminent domain s is high enough that there would be a taking, either purchase or eminent domain could be superior, the latter being possible mainly because under eminent domain there is a possible cost-saving advantage relative to purchase.

Claim (iii), that when there would be a taking under eminent domain, eminent domain is superior to purchase for all n sufficiently high embodies the advantage of eminent domain in the many owner context. This is, in a sense, the central conclusion of the present article. The reasoning is as follows. Under eminent domain, social welfare is $s - cE(v)$, which exceeds $E(v)$ by the hypothesis that there would be a taking under eminent domain. But under the policy of purchase, we know that the probability of successful purchase tends to 0, meaning that social welfare tends to $E(v)$. Hence, if n is sufficiently high, social welfare under purchase must be lower than under eminent domain.

It was also asserted in the claim that not only is eminent domain superior in expected value to purchase for n sufficiently high, but also the probability that the exercise of eminent domain is socially desirable tends to 1. This follows from several

observations. The law of large numbers tells us that, for any $\epsilon > 0$, the probability that the sample mean $(v_1 + \dots + v_n)/n$ lies within ϵ of $E(v)$ tends to 1 with n . Let $\epsilon = .5[s - cE(v) - E(v)]$, which is positive by hypothesis. Hence, the probability that $s - cE(v) > (v_1 + \dots + v_n)/n$ tends to 1 with n . But this means that the probability that a taking is socially desirable tends to 1 with n .

The preceding point and the point that $F(x^*(n))^n$ tends to 0 with n mean that, as n grows large, the likelihood that purchase will result in acquisition of land goes to zero, whereas the likelihood that a taking would be desirable goes to 1. Therefore, the sense in which eminent domain is superior to purchase is, as stated in the Introduction, very strong.

Last, let me remark on the importance of three assumptions to the results in Proposition 2. One assumption is that $c > 0$. This assumption was needed for the limiting results of (b)(ii) and (c)(iii). That is because, if $c = 0$, the $x^*(n)$ do not tend to a limit; they rise unboundedly. In effect, there is no problem with offers failing when n grows because the government can raise its bid x without incurring any social cost. For that reason, when $c = 0$, eminent domain does not become superior to purchase when n grows.

The second assumption is that the distribution of v is unbounded. If v instead had an upper bound b , then $x^*(n)$ might equal b for n sufficiently high, so that the probability of a successful purchase might be 1 rather than tend to 0 with n . However, were that so, it would still be true that eminent domain would be superior to purchase, for eminent domain would involve lower cost to the government ($E(v)$ rather than b).

The third assumption is that v is not observable to the government. If v were observable and the government could perfectly price discriminate, then the policy of purchase would be unambiguously superior to eminent domain regardless of the number of landowners. The proof is as follows. Under purchase, the government's optimal policy would be to make the minimum offer v_i to each of the n landowners if and only if this would raise social welfare, which is to say, if and only if the v_i are such that $s - c(v_1 + \dots + v_n)/n > (v_1 + \dots + v_n)/n$. This policy results in greater social welfare than other pricing policies, and in particular, than offering v_i to each landowner all the time. But the latter policy is equivalent to the use of eminent domain, as it would definitely result in acquisition of the acre and in expected expenditures of $E(v)$.

5. Extensions

5.1 Land Sought By The Government Can Be Located Anywhere

In the above analysis the land that the government sought was a specific parcel. Here, however, I assume that the land sought by the government can be located anywhere in a relevant region. This assumption reflects the common situation in which the government can make roughly equivalent use of differently situated parcels (airports might have approximately comparable value if located within 30 miles of a city and within a few miles of a major highway).

The chief reason that the assumption that the land can be located anywhere is of interest is that, as mentioned in the Introduction, one might think that under it the central result that eminent domain is superior to purchase when the number of landowners grows large would not hold. That is, because the government does not need any *particular* group of landowners to unanimously accept its price offer in order to purchase a parcel of

the kind it seeks, one might believe that the policy of purchase would not tend be dominated by eminent domain when the number of landowners grows. However, this intuition is incorrect. The conclusion that eminent domain is superior to purchase if the number of owners is sufficiently large continues to hold.

To amplify, I assume here that the government needs to obtain at least some square parcel, with a side of length k , where $0 < k \leq 1$; k can be small, and the square parcel can be located anywhere in the square acre of area 1 and can be oriented in any direction. Otherwise, I retain the assumptions from above. I will restrict attention here to proving the limiting result from Proposition 2 (although most of the other results from above carry over as well²¹). The conclusion to be proved is this.

Remark 3. Suppose that the government seeks at least a square parcel of side k , where $1 \geq k > 0$, where the square can be located anywhere in the unit acre and oriented in any direction. Then Proposition 2(c)(iii) holds. Namely, if s exceeds $(1 + c)E(v)$, then for all numbers n of landowners sufficiently high, eminent domain is superior to purchase. Indeed, the probability that eminent domain is socially desirable tends to 1, whereas the probability that purchase would be successful tends to 0.■

Note that this Remark is quite general as it implies that *if the government seeks a parcel that has any contiguous part, the government will tend to fail under the policy of purchase and must use eminent domain if the number of landowners is sufficiently large.* This is because if the parcel sought by the government is required to have any contiguous part of minimum size, it must contain a square of at least k for some positive k .

²¹ It is clear that Proposition 1 and Proposition 2(a) and 2(c)(i)-(ii) would hold by the logic of arguments already made. Proposition 2(b) would hold in modified form (conditions (13) and (15) would be altered).

The strategy of the proof is to demonstrate that for the government to be able to purchase a square of side k is equivalent to its obtaining a type of *run* of successes in Bernoulli trials, where the probability of success in each trial is the probability that its offer of x is accepted by an owner. The reason that purchasing a square is equivalent to a run of successes, rather than to a number of possibly separated successes, is in essence that the owners of the square are neighboring, meaning that they will occur in a row in an ordering of the landowners. Furthermore, the *length* of the run of successes must be such as to cover a certain *proportion* $q > 0$ of all landowners, for the square of side k covers a proportion of the region (of area 1).

From the foregoing argument, we know that for the government to acquire the square it needs, it must achieve a run of successes in n trials of length at least qn . (This, note, is a generalization of the fact that, in section 4 where the government had to acquire a specific parcel, it had to achieve a run of n successes in n trials.) But the probability of such a run tends to 0 as n becomes large. For instance, consider the probability of a run of heads in coin tosses of length at least 10% of all the tosses (q is .1). If the number of tosses is not large, say, 20, then the probability of the run would be high, for the run of heads need only be 2 in length. If, however, the number of tosses is 100,000, the run of heads would have to be 10,000 in length, an extremely unlikely event. I show in the Appendix that the probability of a run of successes of length at least qn tends to 0 as n grows, as long as $q > 0$, where $1 > p > 0$ is the probability of success in a single trial.

Because the probability of a run of successes of the required nature tends to 0 with n , the probability that the policy of purchase results in acquisition of the square of land tends to 0 as n increases, whereas eminent domain by definition can succeed and is

socially desirable, because of the hypothesis that $s > (1 + c)E(v)$. Moreover, the law of large numbers implies, as in Proposition 2, that the probability that a taking is socially desirable approaches 1 as n grows.

Thus, the qualitative result that the policy of purchase fails and that eminent domain is desirable when the number of owners grows sufficiently large holds when the government seeks essentially any kind of contiguous parcel in a region.

5.2 Land Sought By The Government Has Value Proportional To Its Area, Independent Of Location

In section 5.1, although the land sought by the government could be located anywhere in the region, the land had to contain a parcel of at least a minimum size. Suppose here instead that the government places a social value of s per acre that is independent of how much land it acquires or where the land is located. That is, if the government acquires land of area A , the social value of the land is As no matter how small A is. Then it is clear that the problem of maximizing social welfare with respect to each landowner's parcel is independent of the problem with respect to any other owner's parcel. In consequence, the analysis of eminent domain and purchase is exactly that applying in the single owner case of Section 3. We therefore have

Remark 4. Suppose that the social value of land is proportional to its area; the social value equals sA , where A is the area. Then Proposition 1 holds, independently of the number n of landowners. Thus, the optimal use of eminent domain, the optimal offer x^* , and the comparison of eminent domain to purchase, do not depend on n . In particular, if purchase is superior to eminent domain when there is a single landowner, purchase will be superior regardless of the number of owners.■

The difference between this Remark and the previous one is revealed by the resolution of the following apparent paradox. By Remark 4 and the law of large numbers, we know that as the number n of owners of the acre grows, the fraction of the acre that will actually be acquired by the government will tend to $F(x^*) > 0$. Thus, for instance, if $F(x^*)$ is .2, then as n grows, very close to 20% of the land area will be acquired by the government with a probability approaching 1. But by Remark 3, even if the goal of the government were to acquire a much smaller contiguous parcel, say a contiguous parcel covering only .1% of the land area, the government would fail to acquire that relatively tiny parcel with a probability approaching 1 as n grows. The resolution of these two points is that, as n grows, the success of the government in acquiring 20% of the land area must come about through the purchase of many separated parcels.

6. Concluding Comments

Use of eminent domain by private developers. The main point analyzed in this article, that when the number of landowners grows, it becomes more difficult to obtain a parcel through purchase, and that that might justify the use of eminent domain, would seem to apply to private developers of land as well. That is, it might be socially desirable to allow developers to borrow the power of eminent domain when they have to assemble large tracts of land held by many owners, because they would confront a problem in acquiring the required parcels through purchase.²² In reality, the law sometimes does

²² Were one to analyse this situation, one would have to take into account two differences from the situation that I considered. First, the private cost of spending a dollar for the developer is a dollar, instead of the presumably smaller social cost of raising a dollar of funds. This difference between the private and social cost of funds would seem to lead the developer to offer less than the government for land, and thus would apparently exacerbate the problem of failure to obtain land through purchase. Second, the social cost of funds for the developer is zero, since the funds are not raised through taxation, also suggesting that

allow private parties to enjoy the right of eminent domain in roughly these circumstances.²³

Applicability of the analysis to property other than land. The analysis in this article has been interpreted as applying to land, but in what respects does it apply more generally? The single owner analysis of section 3 applies to any type of property; there is no particular reason to view it as bearing only on land. Hence, the point made there that purchase is an advantageous policy of government unless the social value of the property is very high in relation to the distribution of private values applies to property in general, and has a rough consistency to the fact that the law gives the government the right to seize property other than land in certain (mainly emergency) situations.²⁴

The analysis of the multiple owner case, however, is special to land, or at least best fits land. In particular, the assumptions of section 4 – that the government needs all of the parcels making up a specific parcel of land (or the assumption of section 5 that the government needs contiguous parcels) in order to obtain the social benefit from the parcel, and that there is significant variation in the values different owners attach to the land – do not seem natural to make for most other kinds of property. Suppose that the government wants a material, like steel. It would be unlikely that both of the foregoing assumptions applies, namely, that the government’s benefit would be enjoyed mainly only if it obtained at least a threshold quantity of steel, and also (and more importantly)

the circumstances under which acquisition of land by the developer would be desirable would be more broad than by the government.

²³ Notably, the law has granted railroads, utility companies, and, sometimes, private developers of substantial parcels who have a public purpose the right to take land via eminent domain. See, for example, Kelly (2006).

²⁴ This has occurred during wars and other armed conflicts, natural disasters, fires, and various crises. See, for example, Scheiber (2003).

that there would be many owners of that quantity of steel all of whom would have to sell in order for the government to succeed. Hence, the central point of this article, concerning the need for eminent domain in the multiple owner context, seems to bear mainly on land as opposed to other kinds of property.

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Appendix

Proofs of claims not demonstrated in the text are given here.

Proposition 1.

Part (b): The only claim that needs to be shown is that there exists a solution to the government's problem of choosing a price offer $x \geq 0$ to maximize social welfare $W(x)$. To prove existence of a solution, it suffices to demonstrate that there exists a t such that if $x > t$, then $W(x) < W(0)$. For then we know that to maximize $W(x)$, we can restrict attention to x in $[0, t]$, and since $W(x)$ is continuous in x , it must have a maximum in a closed interval such as $[0, t]$. Now let $t = s/c$. Then if $x > t$, social welfare must be negative whenever x is accepted, because the social cost of the purchase will exceed $c(s/c) = s$. Hence, $W(x) < E(v) = W(0)$.

Part (c)(ii): The claim that needs to be shown is that for all s sufficiently high, eminent domain is superior to purchase. This is proved in five steps. Denote the optimal x^* given s by $x^*(s)$.

(1) $x^*(s)$ grows unboundedly with s ; that is, for any $y > 0$, $x^*(s) > y$ for all s sufficiently high: We have that (7), determining $x^*(s)$, may be rewritten as

$$(A1) \quad x = s/(1+c) - [c/(1+c)][F(x)/f(x)].$$

Let $m(y)$ be the maximum of the term $[c/(1+c)][F(x)/f(x)]$ over x in $[0, y]$. I claim that if $s > (y + m(y))(1+c)$, then $x^*(s) > y$. If this is not true, $x^*(s) \leq y$ and (A1) must then be satisfied at some x in $[0, y]$. But if x is in $[0, y]$, the right side of (A1) must exceed

$$(A2) \quad [(y + m(y))(1+c)]/(1+c) - m(y) = y + m(y) - m(y) = y,$$

implying that (A1) cannot be satisfied at x in $[0, y]$, a contradiction.

(2) The probability of purchase tends to 1 as s grows; that is, $\lim_{s \rightarrow \infty} F(x^*(s)) = 1$: This follows immediately from step (1).

(3) For any $y > 0$, first-best social welfare W^* exceeds social welfare under purchase $W(x^*(s))$ by at least cy for s sufficiently high: Social welfare under purchase is obviously lower than W^* by at least the expected social cost of purchase, $F(x^*(s))x^*(s)c$. By step (1), there exists a threshold t_1 such that if $s > t_1$, then $x^*(s) > 2y$ (let $2y$ play the role of y in step (1)). By step (2), there exists a threshold t_2 such that if $s > t_2$, then $F(x^*(s)) > .5$. Let $t = \max(t_1, t_2)$. Then if $s > t$, we know that $F(x^*(s))x^*(s)c > .5(2y)c = cy$, so that W^* exceeds $W(x^*(s))$ by at least cy .

(4) First-best social welfare W^* minus social welfare under eminent domain tends to $cE(v)$ as s grows: For $s > (1+c)E(v)$, we know that a taking is desirable under eminent domain, and let us assume that s is this high. Then W^* minus welfare under eminent domain is

$$(A3) \quad \int_s^{\infty} (v-s)f(v)dv + cE(v),$$

which tends to $cE(v)$ as s grows.

(5) If s is sufficiently high, eminent domain is superior to purchase: This follows from steps (3) and (4). In particular, choose $y = 2E(v)$. Then by step (3), W^* minus welfare under purchase is at least $2cE(v)$ for all s sufficiently large. By step (4), for any positive ϵ , W^* minus welfare under eminent domain is within ϵ of $cE(v)$ for all s

sufficiently large. Hence, choosing ϵ to be small, it follows that for all s high enough, welfare under eminent domain must exceed welfare under purchase. Q.E.D.

Proposition 2.

Part (b)(i): We first need to show that a solution to the problem of choosing the optimal offer exists. This follows from the argument of Proposition 1 that in the problem with one owner an optimal x exists, since again it is clear that the optimal x cannot exceed s/c .

We next must show that $x^*(n) > 0$. This follows because social welfare is higher for any positive $x < s/(1 + c)$ than for $x = 0$: If $x < s/(1 + c)$, then $x < s - cx$. This implies that if n individuals accept x , social welfare must rise ($s - cx$ is social welfare, whereas if individuals keep their land, social welfare is bounded by x). Since the probability of acceptance of an offer is positive for any positive x , social welfare must be higher for any positive $x < s/(1 + c)$ than for $x = 0$.

Part (b)(ii): We first want to show that (15) has a unique solution x^{**} . This is clear, since at $x = 0$, $(s - cx)$ is s and the right side of (15) is 0, and since $(s - cx)$ is decreasing in x and negative for x sufficiently high, whereas the right side is positive and increasing in x .

We next want to show that $\lim_{n \rightarrow \infty} x^*(n) = x^{**}$.²⁵ Let us rewrite (14) and (15) as

$$(A4) \quad 0 = cF(x)/(nf(x)) + x/n + [(n-1)/(nF(x))] \int_0^x vf(v)dv - (s - cx);$$

$$(A5) \quad 0 = F(x) \int_0^x vf(v)dv - (s - cx).$$

Observe first that the right side of (A4) must converge uniformly to the right side of (A5) over any closed interval $[0, t]$. That is, for any $\epsilon > 0$, if n is sufficiently high, the right-hand side of (A4) will be within ϵ of the right side of (A5) for all x in $[0, t]$.²⁶ Let t be s/c , so that the right side of (A4) must converge uniformly to the right side of (A5) in $[0, s/c]$, and recall from the proof of (b)(i), that $x^*(n)$ must be in $[0, s/c]$. Now to establish the claim, it must be shown that for any small $\delta > 0$, $x^*(n)$ will be within δ of x^{**} for all n sufficiently large. We know that over the interval $[0, x^{**} - \delta]$, the minimum distance from 0 of the right side of (A5) is positive, say, b_1 , since (A5) has a unique solution, and similarly over the interval $[x^{**} + \delta, s/c]$, the minimum distance of the right side from 0 is positive, say b_2 . Let $b = \min(b_1, b_2)$, so that the distance of (A5) from 0 over the two intervals is at least b . Since the right side of (A4) converges uniformly to the right side of

²⁵ If $x^*(n)$ is not unique, choose any optimal $x^*(n)$ as the element in the sequence of $x^*(n)$.

²⁶ The difference between the right sides of (A4) and (A5) is

$$cF(x)/(nf(x)) + x/n - \int_0^x vf(v)dv/(nF(x)) = cF(x)/(nf(x)) + x/n - E[v | v \leq x]/n,$$

which is of the form $g(x)/n$, where $g(x)$ is continuous. Since $|g(x)|$ has a maximum M over $[0, t]$, the distance between the right sides of (A4) and (A5) on $[0, t]$ is bounded by M/n . Hence, if $n > M/\epsilon$, the distance is bounded by $M/(M/\epsilon) = \epsilon$.

(A5), if we choose $\epsilon = b/2$, we know that for n sufficiently large, the right side of (A4) must be within $b/2$ of (A5) in $[0, s/c]$. But this implies that the right side of (A4) cannot equal 0 in $[0, x^{**} - \delta]$ or in $[x^{**} + \delta, s/c]$. Hence, the right side of (A4) can equal 0 only within δ of x^{**} , which is to say, $x^*(n)$ must be within δ of x^{**} .

Now let us demonstrate that $x^{**} > x^*(1)$. To do this, it suffices to show that the left side of (15), $(s - cx)$, exceeds the right side when evaluated at $x^*(1)$, since as noted the left side is decreasing in x and the right side is increasing in x . Now from (6), which determines $x^*(1)$, we have

$$(A6) \quad s - cx = cF(x)/f(x) + x.$$

But

$$(A7) \quad cF(x)/f(x) + x > x = xF(x)/F(x) > \int_0^x vf(v)dv/F(x) > F(x) \int_0^x vf(v)dv,$$

which is the right side of (15).

We next show that $x^{**} > s$ for c sufficiently low. To prove this, it suffices to demonstrate that the left side of (15) exceeds the right side when evaluated at $x = s$ for c sufficiently low. We know that the right side of (15) at s is less than $F(s)s < s$, and thus the left-hand side $s - cs$ must exceed the right-hand side for all c sufficiently low.

Finally, that the probability of successful purchase $F(x^*(n))^n$ tends to 0 is clear since $x^*(n)$ tends to a limit x^{**} , and $F(x^{**}) < 1$.

Part (c)(i): That purchase is superior to eminent domain if $s \leq (1 + c)E(v)$, follows from the result (b)(i) that $x^*(n) > 0$, for then $W_n(x^*(n)) > W_n(0) = E(v)$, which is social welfare under eminent domain given the hypothesis about s .

Part (c)(ii): To show that if $s > (1 + c)E(v)$, purchase is superior to eminent domain for c sufficiently low, it suffices to demonstrate that purchase is superior to eminent domain when $c = 0$, since $W_n(x^*(n))$ is continuous in c . When $c = 0$, we have (see (13))

$$(A8) \quad W_n(x) = F(x)^{n-2}f(x)[F(x)(ns - x) - (n - 1) \int_0^x vf(v)dv],$$

from which it is clear that $W_n(x)$ is decreasing for $x \geq ns$. Also, when $c = 0$,

$$(A9) \quad \lim_{x \rightarrow \infty} W_n(x) = \lim_{x \rightarrow \infty} \{F(x)^n s + E(v) - F(x)^{n-1} \int_0^x vf(v)dv\} = s.$$

Hence, $W_n(ns) > s$, and thus $W_n(x^*(n)) > s$. But s is social welfare under eminent domain. In other words, social welfare is higher under purchase than under eminent domain when $c = 0$.

To prove that eminent domain is superior to purchase for all s sufficiently high, we proceed in five steps; the proof is analogous to that in Proposition 1.

(1) $x^*(s)$ grows unboundedly with s ; that is, for any $y > 0$, $x^*(s) > y$ for s sufficiently high: Choose some positive s , say s_0 . Now to prove that $x^*(s) > y$ for s sufficiently high, it suffices to prove only that $x^*(s)$ cannot lie in $[x^*(s_0), y]$ for s sufficiently high. This will suffice because we already know that $x^*(s) > x^*(s_0)$ for $s > s_0$,

since $x^*(s)$ is increasing in s .²⁷ To prove that $x^*(s)$ cannot be in $[x^*(s_0), y]$ for s sufficiently high, we show that (13) cannot be satisfied in $[x^*(s_0), y]$ if s is sufficiently high. To do this, we demonstrate that

$$(A10) \quad F(x)^{n-1}f(x)[n(s-cx)-x] - cF(x)^n - (n-1)F(x)^{n-2}f(x)\int_0^x vf(v)dv > 0$$

for x in $[x^*(s_0), y]$ if s is sufficiently high. This is true since the first term of (A10) grows unboundedly with s . In particular, let m be the minimum of $F(x)^{n-1}f(x)$ over $[x^*(s_0), y]$. Thus, for x in $[x^*(s_0), y]$, the first term is at least $m[n(s-cy)-y]$, which grows unboundedly with s . The other terms do not depend on s so must be exceeded by the first term for s sufficiently high.

(2) The probability of purchase tends to 1 as s grows; that is, $\lim_{s \rightarrow \infty} F(x^*(s))^n = 1$: This follows from step (1).

(3) For any positive y , first-best social welfare W^* exceeds social welfare under purchase by at least cy for all s sufficiently high: Social welfare under purchase is lower than W^* by at least the expected social cost of purchase, $F(x^*(s))^n cx^*(s)$. By step (1), there exists a threshold t_1 such that if $s > t_1$, then $x^*(s) > 2y$. By step (2), there exists a threshold t_2 such that if $s > t_2$, then $F(x^*(s))^n > .5$. Let $t = \max(t_1, t_2)$. Then if $s > t$, we know that $F(x^*(s))^n cx^*(s) > .5c(2y) = cy$, so that W^* exceeds social welfare under purchase by at least cy .

(4) First-best social welfare minus social welfare under eminent domain tends to $cE(v)$ as s grows: For all s above $(1+c)E(v)$, we know that a taking is desirable under eminent domain, and let us assume that s is this high. Then W^* minus welfare under eminent domain is

$$(A11) \quad \int_0^\infty \dots \int_0^\infty \int_{s-(v_2+\dots+v_n)}^\infty [(v_1/n + \dots + v_n/n) - s] f(v_1)f(v_2)\dots f(v_n) dv_1 dv_2 \dots dv_n + cE(v).$$

The first term is the integral of the social loss when there is a socially undesirable taking, that is, when $v_1/n + \dots + v_n/n \geq s$. This term tends to 0 as s grows. Hence W^* minus welfare under eminent domain tends to the cost of compensation $cE(v)$ as s grows.

(5) If s is sufficiently high, eminent domain is superior to purchase: This follows from steps (3) and (4). In particular, choose $y = 2E(v)$. Then by step (3), W^* minus social welfare under purchase is at least $2cE(v)$ for all s sufficiently large. By step (4), for any positive ϵ , W^* minus welfare under eminent domain is within ϵ of $cE(v)$ for all s sufficiently large. Hence, choosing ϵ to be small, it follows that for all s high enough, welfare under eminent domain must exceed welfare under purchase.

Part (c)(iii): We need to prove that if s exceeds $(1+c)E(v)$, then for all n sufficiently high, eminent domain is superior to purchase. Social welfare under purchase is

$$(A12) \quad W_n(x^*(n)) = F(x^*(n))^n (s - cx^*(n)) + E(v) - F(x^*(n))^{n-1} \int_0^{x^*(n)} vf(v)dv.$$

²⁷ We know that $x^{*'}(s) > 0$ because the partial derivative of the left side of (13) with respect to s is positive.

Since $x^*(n)$ tends to x^{**} as n grows by part (b), $W_n(x^*(n))$ tends to $E(v)$. However, social welfare under eminent domain is $s - cE(v) > E(v)$. Hence, if n is sufficiently high, social welfare under eminent domain exceeds social welfare under purchase. Q.E.D.

Remark 3.

The sequence of increasing numbers of owners of land will for convenience be taken to be a sequence of successively finer partitions of the unit acre constructed as follows. The first partition in the sequence is obtained by dividing the unit square in half by a line going down its middle. Thus, the first partition has two partition elements (two parcels held by different owners), each a rectangle that is $1/2$ in width and 1 in length. The second partition is obtained by taking each of the two rectangles in the first partition and dividing it into two squares by drawing a line horizontally across it. Thus, the second partition has four elements, each a square with side $1/2$. Continuing in this way, we obtain partitions that are successively finer, where the following is true. The n^{th} partition, has 2^n partition elements, each with area $1/2^n$. In every odd numbered partition, the elements are rectangles with sides of length $1/2^{(n+1)/2}$ and $1/2^{(n-1)/2}$; and in every even numbered partition, the elements are squares with sides of length $1/2^{n/2}$.

The proof consists of five of steps.

(1) Let n be the number of repeated independent trials, in each of which there are two possible outcomes, success or failure, and in which the probability of success in a trial is p , where $1 > p > 0$. Let q be a proportion, where $1 > q > 0$. Let $P(n)$ be the probability of a run of consecutive successes of at least qn in length. Then $\lim_{n \rightarrow \infty} P(n) = 0$: Denote by $i(qn)$ be the lowest integer greater than or equal to qn , so that the run of successes must be at least $i(qn)$ in length. The probability that there is a run of $i(qn)$ successes beginning in the first trial, and allowing for any outcomes (including a longer run) otherwise is $p^{i(qn)}$. Similarly, the probability that there is a run of $i(qn)$ successes stretching from the second trial up to trial $i(qn) + 1$, and allowing for any outcomes otherwise (including a longer run beginning earlier or ending later) is also $p^{i(qn)}$. Because there are $n - i(qn) + 1$ such originating points for runs of at least length $i(qn)$, the probability $P(n)$ of a run of at least length $i(qn)$ must be less than $(n - i(qn) + 1)p^{i(qn)}$: for the $n - i(qn) + 1$ events under discussion not only include all the ways in which there could be a run of length at least $i(qn)$, but also the events are not mutually exclusive. (For example, a run of length $2i(qn)$ beginning at trial 1 would be included in the first event, the second, ..., as well as the $i(qn)^{\text{th}}$.) Now let $T(n) = (n - i(qn) + 1)p^{i(qn)}$. Then (A13) $T(n + 1)/T(n) = [(n + 1 - i(q(n + 1)) + 1)/(n - i(qn) + 1)][p^{i(q(n+1))}/p^{i(qn)}]$. It is clear that the first term in brackets tends to 1 as n grows. Observe that $i(q(n + 1))$ equals either $i(qn)$ or $i(qn) + 1$. Hence, the second term in brackets is either 1 or p , where p must occur with regularity, since $i(q(n + 1))$ will be 1 higher than $i(qn)$ with a frequency of about $1/q$. This implies that, for high n , the sequence of ratios $T(n + 1)/T(n)$ is approximately a sequence of 1's and p 's, where p occurs with a constant frequency. It follows that $T(n)$ tends to 0 with n and thus that $P(n)$, being less than $T(n)$ tends to 0 with n .

(2) For the government to purchase a square parcel of side k when it makes an offer of x to owners in the n^{th} partition, it must achieve a run of successes of at least $q2^n$ in length in 2^n independent trials, where the probability of success in a trial is $F(x)$ and

where $q > 0$ and is independent of n : Suppose that the acre is partitioned into squares of side no more than $k/4$. Then it is clear from geometry that if the government acquires a square of side k , it must entirely contain some square of side at most $k/4$. Let n_0 be the first even numbered partition with squares of side less than or equal to $k/4$. Thus, if the government acquires its parcel, some square in the n_0 partition must be entirely contained in that parcel.

In each partition n beyond the n_0^{th} , for each square in the n_0 partition, number consecutively all owners who have land in that square. For instance, in partition $n_0 + 4$, each of the squares in the n_0 partition will be divided into 16 smaller squares. The owners of these 16 squares making up a square of the n_0 partition are to be numbered consecutively. (The numbering of owners whose land falls in different squares of the n_0 partition does not matter.)

We know, therefore, that, for any partition n beyond the n_0^{th} , if the government acquires its square parcel of side k by making a price offer of x , the government must have succeeded in acquiring a run of parcels covering an entire square of the n_0 partition, where the probability of success for each parcel is $F(x)$. Since such a square in the n_0 partition has area $1/2^{n_0}$, the run must cover that area, so the length of the run must be $(1/2^{n_0})2^n$, which is to say that $q = 1/2^{n_0}$.

(3) The optimal price offer for any partition n , denoted $x^*(n)$, of the government must be below s/c : If the offer is s/c or above, then social welfare must be 0 or below if land is acquired, as the social cost of making payment is at least $c(s/c) = s$. Thus, such an offer cannot be optimal.

(4) The probability that the government succeeds in obtaining its parcel when it makes optimal price offers $x^*(n)$ tends to 0 as the number n of partitions and of owners grows large: Let $Q(n)$ be the probability that the government succeeds in obtaining its parcel with respect to partition n of the acre when it makes the optimal offer $x^*(n)$. Let $P(n)$ be the probability that the government succeeds in obtaining its parcel with respect to partition n of the acre if it makes the offer s/c . By step (3), $x^*(n) < s/c$, so that $Q(n) < P(n)$. By steps (1) and (2), $\lim_{n \rightarrow \infty} P(n) = 0$. Since $Q(n) < P(n)$, $\lim_{n \rightarrow \infty} Q(n) = 0$.

(5) The claimed results now follow for the reasons that Proposition 2(c)(iii) does. Namely, since the probability $Q(n)$ that the government obtains its parcel tends to 0, social welfare under purchase tends to $E(v)$. But since social welfare under eminent domain is $s - cE(v) > E(v)$, if the partition n is sufficiently high, social welfare under eminent domain exceeds social welfare under purchase. Also, the reason that the probability that government acquisition of a parcel of the type it seeks is socially desirable goes to 1 with n is essentially that, as discussed after Proposition 2, the sample mean must tend in probability to $E(v)$ by the law of large numbers. Q.E.D.

Remark 4.

This is self-evident from the text.