

Preferences for Risk in a Dynamic Model with Consumption Commitments

Galina Vereshchagina*
University of Iowa

This draft: March 2007

Abstract

This paper characterizes the solution to the consumer's dynamic decision problem in the presence of *consumption commitments* (goods that involve transaction costs and are infrequently adjusted). The findings in recent theoretical literature have suggested that consumption commitments amplify risk aversion in static models (Postlewaite et. el. 2006) and in a dynamic model without the borrowing constraints and with zero interest rate (Chetty and Szeidl 2007).

This paper illustrates that the opposite result might obtain in a more general dynamic environment. We show that, if the price of a risk-free bond does not exceed the time preference rate (as it is usually predicted by the general equilibrium macroeconomic models), the consumers who start saving in order to increase consumption of the commitment good in the future become risk neutral or risk lovers. We argue that such behavior is likely to arise either because the interest rate is positive or because the consumers cannot borrow against their future income. The latter observation suggests that in the economies with consumption commitments borrowing constraints can make uninsured risk desirable (in contrast, it is known that in standard models borrowing constraints increase the cost of uninsured risk).

*Contact address: Department of Economics, University of Iowa, 21 E Market St., Iowa City, IA 52242, email: galina-vereshchagina@uiowa.edu

1 Introduction

Consumption commitments arise when some goods involve substantial transaction costs and thus are infrequently adjusted. Typical examples of such goods are housing, land, vehicles, etc. Consumption commitments have attracted attention in recent economic literature since it has been documented that they constitute a large share in households' life-time expenses¹. Various effects of commitment goods have been discussed; special attention being paid to their impact on consumers' risk preferences (e.g. Chetty and Szeidl 2007, Shore and Sinai 2005, Postlewaite et. al. 2006, etc.). It has been argued that the presence of consumption commitments increases risk aversion of individuals.

Intuitively, if consumption of some of the goods remains unchanged in response to (relatively small) permanent changes in current wealth or income, consumption of the remaining (flexible) goods would vary "too much". Such extra volatility of these goods raises the welfare cost of risk compared to the environment in which all the goods could be adjusted flexibly. Thus it has been suggested that consumption commitments magnify risk aversion. So far, this result had been rigorously established in static models (Postlewaite et. el. 2006) and in a dynamic model under some specific assumptions discussed later (Chetty and Szeidl 2007).

This paper argues that in a more general environment the effect of consumption commitments on consumers' risk preferences might be very different; the consumers with particular wealth levels might even become risk lovers. As in the previous studies, we emphasize that consumers' attitudes towards risk are determined not by the properties of their instantaneous utility function – but by the shape of their indirect life-time utility function arising from the solution of a dynamic optimization problem with commitments. In our model (as well as in the previous literature), the indirect life-time utility is a function of consumers' wealth. We show that for the consumers with particular wealth levels this function may be concave, linear or even convex, depending on the relative values of the interest rate and the time discount factor.

To briefly outline the driving forces of our results, consider a case when the gross

¹Warren and Tyagi (2003) find that a typical American family earmarks 75% of their income on "fixed expenses" such as mortgage, car payments, etc. Chetty and Szeidl (2007) use CES data and estimate that around 50% of households' life-time wealth is spent on "commitment goods".

return to risk-free savings is smaller than the reverse of the time discount factor ($\beta(1+r) < 1$), as it is usually predicted by the general equilibrium macro models with uninsured risk (e.g. Aiyagari 1994). Suppose also that the agent has committed to little housing expenses in the beginning of his life, but is currently accumulating wealth in order to move to a bigger house in the future. As long as this agent stays in a small house, his consumption of a flexible good has the standard properties obtained in a neoclassical growth model. In particular, the current marginal utility from consuming the flexible good is equalized with the discounted future marginal utility times the gross rate of return ($u'(c_t) = \beta(1+r)u'(c_{t+1})$); thus the flexible good's consumption declines over time. At the same time, by the envelope condition, the consumption level of the flexible good determines the slope of the value function (indirect life-time utility function): as consumption declines, the value function gets steeper. Since consumption declines simultaneously with an increase in wealth (recall that the consumer makes savings in order to move to a bigger house), the value function (as a function of wealth) gets steeper as wealth rises, thereby generating demand for wealth lotteries.

Why would some consumers decide to commit to a low level of housing consumption in the beginning of their life and then start accumulating wealth in order to move to a more expensive housing in the future? One possibility could be the presence of the borrowing constraints. If buying a small house requires lower adjustment cost than purchasing a big house, if consumers have little wealth in the beginning of their life and cannot borrow against their future (possibly large) income, they would have no other choice but to move into a small house and start accumulating wealth in order to pay bigger adjustment cost and eventually switch to a bigger house. While they are living in a small house, their value function has the properties described in the previous paragraph and thus these consumers could actually become risk lovers. Therefore, the presence of commitment goods has a somewhat surprising impact on the role of borrowing constraints. It is commonly believed that borrowing constraints make risk more costly (because people have to give up their consumption when they are hit with the temporary negative shocks). This paper points out to a situation in which borrowing constraints have a reverse effect: they generate an interval of wealth levels, within which consumers are actually willing to pay for taking a lottery; though outside of this interval borrowing constraints have a standard effect by raising consumers' risk aversion.

Previous literature has discussed a number of situations in which risk loving behavior might arise endogenously. Most of them are various version of discrete choice models which generate kink in the indirect utility function. For example, Vereshchagina and Hopenhayn (2006) model a discrete occupational choice model and argue that entrepreneurs might be willing to take excess risk because their value function has a kink at a wealth level at which agents are indifferent between being workers and being entrepreneurs. Albuquerque and Hopenhayn (2004) show that risk taking might be the feature of the optimal borrowing contract between the bank and a risk-neutral firm; such risk taking is used to eliminate the kink occurring at the intersection of the firm's endogenous value and its exogenous liquidation cost. Athreya (2002) argues that poor consumers might have additional incentives to make risky investment and are more likely to default because their value has a kink at a point at which the value of default intersects the value of no-default. Even Chetty and Szeidl (2007) acknowledge that consumption commitments can generate demand for moderate-stake risks due to local convexity of the value function at a point at which the consumer is indifferent between staying in an house and adjusting his housing consumption to a new level.

In contrast to all these papers, we describe how the local convexity of the indirect utility function arises not due to a presence of a kink, but as an outcome of optimal saving behavior of consumers either with little patience or facing low interest rate (so that $\beta(1+r)$ is sufficiently low). The resulting value function is convex not at one single point at which the consumer is indifferent between two discrete options; instead, it is convex within an interval of wealth levels. Therefore this project brings up attention to a (hopefully) novel relationship between agents' patience, saving opportunities, borrowing constraints and risk preferences.

The paper is organized as follows. Section 2 analyzes a simple model in which the choice of housing is exogenously discrete. It describes the methodology used to characterize the consumers' decision problem and emphasizes the role of $\beta(1+r)$ in shaping the consumers' risk attitudes. It also contrasts our model with Chetty and Szeidl (2007) and discusses why the effects of consumption commitments on risk preferences studied in our paper could not arise in their environment. Section 3 extends our benchmark model by allowing for more flexible housing choice and argues that either the positive interest rate or the borrowing constraints alone might lead to risk loving (or risk neutral) behavior in the presence of consumption commitments.

2 The Model: Discrete Choice of Housing

2.1 Setup

We will illustrate our main results in a discrete-time infinite horizon model. Assume that a consumer receives income $y \geq 0$ in every period and can spend it on food and housing. The consumer's life-time utility is given by:

$$\sum_{t=0}^{+\infty} \beta^t u(c_t, h_t),$$

where $\beta \in (0, 1)$ is the time discount factor, and the instantaneous utility in period t is derived from the consumption of food c_t and housing h_t . Assume that $u(\cdot, h)$ is defined on $(0, +\infty)$ and is strictly increasing, strictly concave, bounded from above and satisfies Inada conditions.

We assume that food consumption is flexible and can be adjusted at no cost. In contrast, the choice of housing consumption is discrete, $h_t \in \{h^1, h^2\}$ (where $0 < h^1 < h^2$) and that the adjustment cost $\eta > 0$ must be paid if the agent switches from one housing level to another.² We also impose the following simplifying assumptions about the dynamics of the housing: (i) the agent's initial housing level is $h_0 = h^1$ and (ii) the agent can adjust the level of housing only once (e.g., he cannot move from h^2 to h^1). The former assumption is crucial for obtaining the main results of the paper (as we will show later, only the consumers who are planning to switch from low to high level of housing might become risk lovers).³ The latter assumption significantly simplifies the analysis; it implies that the life-time utility of the consumer who has already switched to high housing consumption can be easily determined. In the final Section of the paper we argue that our results should still hold even if the second assumption is relaxed, but we do not formally analyze this case. Finally, it is worth pointing out that even under these simplifying assumption our model is capable of producing richer housing dynamics than the model of Chetty and Szeidl (2007).

²In the extension of the model in the next Section we allow for a flexible choice of housing after the switching cost η is paid. All the main results of this section would also apply in that environment.

³Such assumption could be justified by interpreting η as a cost of moving into h^2 and assuming that it is costless to move into h^1 .

Finally, assume that the consumer can save in a risk-free bond which offers interest rate r and he might be facing the borrowing constraints preventing his asset holdings a_t fall below some exogenous level \underline{a} . Thus the consumer's decision problem can be written as the choice of the food consumption and wealth profiles $\{c_t, a_t\}_{t=0}^{+\infty}$ as well as the moment of switching from low to high housing consumption:

$$\begin{aligned}
& \max_{\{c_t, a_t\}, T \in \{0, 1, 2, \dots, +\infty\}} \sum_{t=0}^{T-1} \beta^t (u(c_t) + v^1) + \sum_{t=T}^{+\infty} \beta^t (u(c_t) + v^2) \\
& \text{s.t. } c_t + h^1 + \frac{a_{t+1}}{1+r} \leq a_t + y, \quad 0 \leq t < T, \\
& \quad c_t + h^2 + \eta^1 + \frac{a_{t+1}}{1+r} \leq a_t + y, \quad t = T, \\
& \quad c_t + h^2 + \frac{a_{t+1}}{1+r} \leq a_t + y, \quad t > T, \\
& \quad a_t \geq \underline{a}, \quad t \geq 0, \\
& \quad a_0 \text{ is given.}
\end{aligned} \tag{1}$$

Note that the agent might decide to switch to h^2 right away by setting $T = 0$ (in which case the first budget constraint is irrelevant) or to remain in a small house h^1 forever by setting $T = +\infty$.

2.2 Solution

We can reformulate the consumer's decision problem (1) recursively. The value of the agent who has already moved to a house h^2 can be found from the following dynamic programming problem:

$$\begin{aligned}
V^2(a) &= \max_{c, a' > \underline{a}} \{u(c, h^2) + \beta V^2(a')\} \\
& \text{s.t. } c + h^2 + \frac{a'}{1+r} \leq a + y.
\end{aligned} \tag{2}$$

This consumer remains in the big house h^2 for the rest of his life, and thus his next-period value is given by $v^2(a')$. If the consumer has not moved to the big house yet, in the current period he might choose whether to remain in the old house or to move into a new house and pay the cost η . Therefore, his value can be found as

$$V(a) = \max\{V^1(a), V^2(a - \eta)\}, \tag{3}$$

where

$$V^1(a) = \max_{c, a' > \underline{a}} \{u(c, h^1) + \beta V(a')\} \quad (4)$$

$$s.t. \quad c + h^1 + \frac{a'}{1+r} \leq a + y$$

Note that if the consumer decides to remain in the old house, he would still have a choice of moving into a new house in the future, thus his continuation value is given by $V(a')$.

Alternatively, we can represent the decision problem of the agent who has not moved to a new house yet as the choice of the moment of switching

$$V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), V_3(a), \dots\}, \quad (5)$$

where $V_t(a)$ stands for the value of the consumer who plans to move into a new house in t periods. Thus

$$V_0(a) = V^2(a - \eta) \quad (6)$$

and the rest of the sequence $\{V_t(a)\}_{t=1}^{+\infty}$ can be found recursively:

$$V_{t+1}(a) = \max_{a' > \underline{a}} \{u(a + y - h^1 - \frac{a'}{1+r}, h^1) + \beta V_t(a')\} = TV_t(a). \quad (7)$$

Obviously, $V_\infty(a)$ is the fixed point of the operator T :

$$V_\infty(a) = \max_{a' > \underline{a}} \{u(a + y - h^1 - \frac{a'}{1+r}, h^1) + \beta V_\infty(a')\} = \lim_{t \rightarrow +\infty} T^t V_0(a) \quad (8)$$

Such recursive representation helps to fully characterize the solution to the consumer's decision problem (1) and describe the properties of the value function $V(a)$ over its domain $(\underline{a}, +\infty)$.

The crucial step in characterizing $V(a)$ is establishing the following Lemma:

Lemma 1

Suppose that $V_0(a)$ and $V_1(a)$ are concave, increasing, continuously differentiable and bounded from above functions. Assume that $V_0(a)$ and $V_1(a)$ have at most one intersection and $V_0(a) > V_1(a)$ for all $a > a_1^$. Then*

(i) $TV_0(a)$ and $TV_1(a)$ cannot have more than one intersection and $TV_0(a) >$

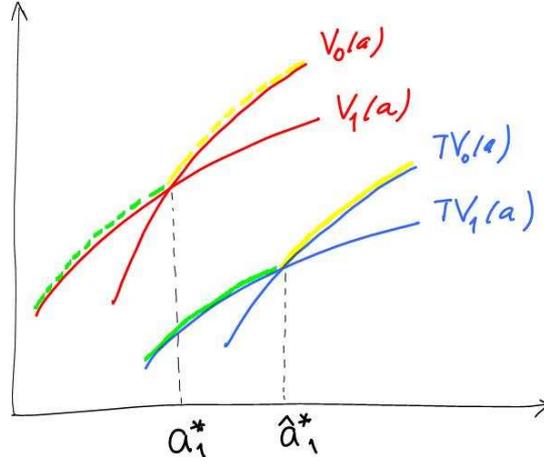


Figure 1: Illustration to Lemma

$TV_1(a)$ for sufficiently large a ; let $\hat{a}_1^* = \min\{a : TV_0(a) \geq TV_1(a)\}$;

- (ii) $a'_0(a) > a_1^*$ for all $a \geq \hat{a}_1^*$ and $a'_1(a) < a_1^*$ for all $a \leq \hat{a}_1^*$, where $a'_i(a)$ the optimal saving policy of the agent maximizing $TV_i(a)$, $i = 0, 1$.

Note that Lemma 1 holds for any $V_0(a)$ and $V_1(a)$ which satisfy its conditions. We later on will verify that these conditions hold for the corresponding value functions in our model. Also note that Lemma 1 does not require the difference between the two value functions, $V_0(a) - V_1(a)$, to be monotone.

The results of Lemma 1 are very intuitive. The first statement says that if the choice between the two options $V_0(a)$ and $V_1(a)$ is described by a unique threshold rule then the decision problem of the agent who chooses whether to continue with $V_0(a)$ or $V_1(a)$ in the future is also described by the unique threshold rule for the agent's current wealth level. The second statement says that the savings policy of the agent is consistent with his future choice between $V_0(a)$ and $V_1(a)$: for instance, if the agent prefers $TV_0(a)$ in the current period then he would save so much that in the next period he would indeed prefer to choose $V_0(a')$. The proof of Lemma 1 is in the Appendix.

We can now use Lemma 1 to provide a complete characterization of $V(a)$. The first statement of Lemma 1 implies that if $V_0(a)$ and $V_1(a)$ have a unique intersection then any two consecutive functions $V_t(a)$ and $V_{t+1}(a)$ from the sequence $\{V_t(a)\}_{t=1}^{+\infty}$ defined by (7) would also have at most one intersection. The second statement of the Lemma 1 is used to argue that the corresponding sequence of the cutoff levels

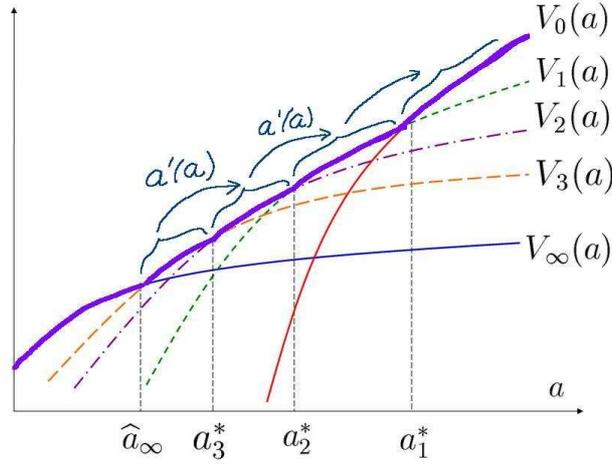


Figure 2: An example of the function $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), V_2(a), \dots\}$ (highlighted in violet) and the optimal savings policy $a'(a)$ for the agent with $h_0 = h^1$.

is monotonically decreasing as long as $V_{t+1}(a) > \max\{V_\infty(a), V_0(a), V_1(a), \dots, V_t(a)\}$ for some $a \geq \underline{a}$. Thus we can formulate the following Proposition:

PROPOSITION 1 (*Characterization of $V(a)$*)

Suppose that the following conditions are satisfied

- (a) $V_0(a)$ and $V_\infty(a)$ are concave, increasing and continuously differentiable;
- (b) $TV_0(a)$ and $V_0(a)$ have at most one intersection and $TV_0(a) < V_0(a)$ for sufficiently high a ;
- (c) $V_\infty(a)$ and $V_0(a)$ at most one intersection and $V_\infty(a) < V_0(a)$ for sufficiently high a (needed only if $\beta(1+r) \leq 1$).

Then there exists $\underline{a} \leq \hat{a}_\infty \leq a_1^*$ such that the agent

- (i) stays in house h^1 forever if $a < \hat{a}_\infty$;
- (ii) switches to house h^2 right away if $a \geq a_1^*$;
- (iii) stays in h^1 for $T(a)$ periods and then switches to h^2 if $a \in [\hat{a}_\infty, a_1^*]$; $T(a)$ is decreasing in a .

Figure 2 illustrates an example of the value function $V(a)$ characterized in Proposition 1. The agent decides to postpone switching to a new house by three periods or less if his wealth falls into the intermediate interval $(\widehat{a}_\infty, a_1^*)$. The second statement of Lemma 1 implies that within this intermediate interval the consumer's wealth is growing over time: for instance, if the agent's current wealth falls into $[a_3^*, a_2^*)$ and he currently chooses to remain in the old house for two more periods, the agent would save so much that in the following period his wealth would fall into $[a_2^*, a_1^*)$ and he would be planning to remain in the old house for one period only.

In order to apply Proposition 1 to our model, we remain to verify that $V_0(a)$ and $V_\infty(a)$ defined in (6) and (8) respectively indeed satisfy the conditions (a)-(c) of the Proposition 1. The standard dynamic programming arguments imply that both value functions inherit the properties of $u(\cdot, h)$, thus (a) of Proposition 1 holds. The last condition (c) is trivially verified if we assume that $u_1(c, h^1) \leq u_1(c, h^2)$ for all c (i.e. $u_{12}(c, h) \geq 0$). However, verifying the second condition (b) is more cumbersome due to the presence of the borrowing constraint $a \geq \underline{a}$. We do it by establishing the following additional result:

Lemma 2 (*Single-crossing of $V_0(a)$ and $TV_0(a)$*)

If $V_0'(a) \leq TV_0'(a)$ then $V_0(a) > TV_0(a)$, and thus $V_0(a)$ and $TV_0(a)$ might have only one intersection at which $V_0'(a) < TV_0'(a)$.

The dynamics of wealth inside the intermediate interval $(\widehat{a}_\infty, a_1^*)$ helps to describe risk preferences of the consumers who are planning to switch to a new house in the future. This is done in the following section.

2.3 The Role of $\beta(1+r)$ for Risk Preferences

Let us look more closely at the shape of $V(a)$ inside the interval $(\widehat{a}_\infty, a_1^*)$ (consider the case when $V(a) > \max\{V_\infty(a), V_0(a)\}$ for some $a > \widehat{a}_\infty$). First, notice that the envelope condition to (7) implies that

$$V'(a) = V'_{t+1}(a) = u'(c(a)) \text{ for all } a \in [a_{t+1}^*, a_t^*), 0 \leq t \leq T, \quad (9)$$

where $c(a)$ is the optimal food consumption of the agent with current wealth a . The first order conditions to (7), which hold with equality since consumers' savings are

interior, imply that

$$u'(c(a)) = \beta(1+r)V'_t(a'(a)) \text{ for all } a \in [a_{t+1}^*, a_t^*), 0 \leq t \leq T. \quad (10)$$

Recalling that $a'(a) \in [a_t^*, a_{t-1}^*)$ and thus $V_t(a'(a)) = V(a'(a))$, we obtain that

$$V'(a) = \beta(1+r)V'(a'(a)) \text{ for all } a \in [a_{t+1}^*, a_t^*), 0 \leq t \leq T. \quad (11)$$

This implies that if $\beta(1+r) < 1$ then the consumer's value function gets steeper while his wealth rises over time.

Figure 3 illustrates the dynamics of the consumer's value and wealth starting from some $a_0 \in (a_\infty^*, a_T^*)$. Relationship (11) implies that

$$V'(a_0) < V'(a_1) < \dots < V'(a_T) \quad \text{and} \quad a_0 < a_1 < \dots < a_T.$$

This creates some sort of convexity in the indirect utility function within the interval (a_∞^*, a_T^*) . In particular, at time periods $t = 1 \dots T-1$ the consumer would like to take a lottery randomizing between a_{t-1} and a_{t+1} with the expected payoff a_t . In general, this consumer would be better off if he is able to randomize over any combination of the wealth levels $\{a_0, a_1, \dots, a_T\}$ experienced during the first T periods of his life if the lottery's expected payoff is equal to his current wealth level.

Note also that, due to time discreteness, the consumer would not necessarily be willing to take any fair lottery with payoffs from (a_∞^*, a_T^*) ; it is important that the outcomes of the lottery belong to the set $\{a_0, a_1, \dots, a_T\}$. This is because the consumer's indirect life-time utility is strictly concave within each interval (a_{t+1}^*, a_t^*) , $0 \leq t \leq T$. However, all above arguments do not depend on the length of the time period. Thus we can always rewrite the model by shortening the time period and, thereby, allowing consumers to make adjustments more frequently (doing this would only make the model more realistic).⁴ Under such modification, the intervals within which $V(a)$ is strictly concave would shrink. Intuition suggests that as the length of the period converges to zero (i.e. the model becomes a continuous time model), the consumer's indirect utility function would be strictly convex in the

⁴When we make the time period shorter, we need to adjust the model's parameters correspondingly. For example, if we split each period into n equal periods, we need to set $\hat{\beta} = \beta^{1/n}$, $\hat{r} = (1+r)^{1/n} - 1$ and $\hat{y} = \frac{1+r}{r} \frac{\hat{r}}{1+\hat{r}} y$.

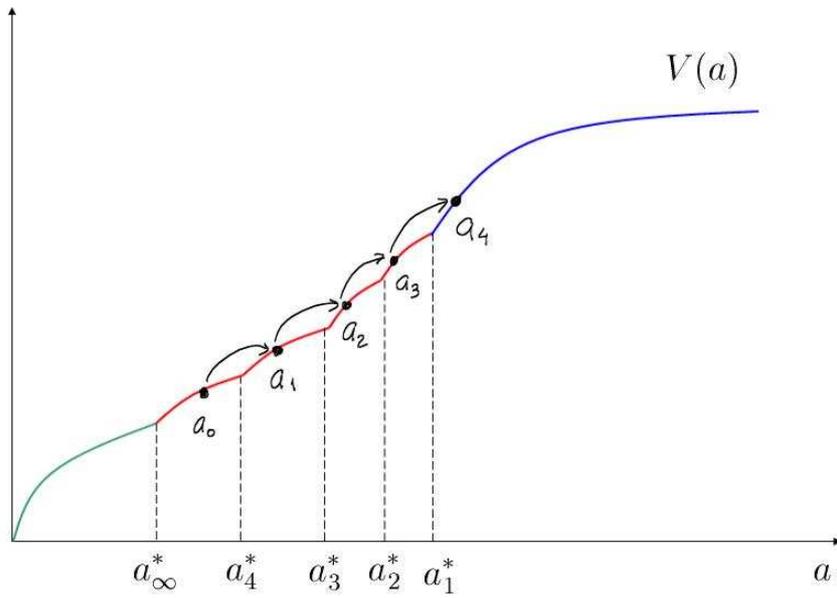


Figure 3: The effect of consumption commitments on the agents' indirect utility, $\beta(1+r) < 1$

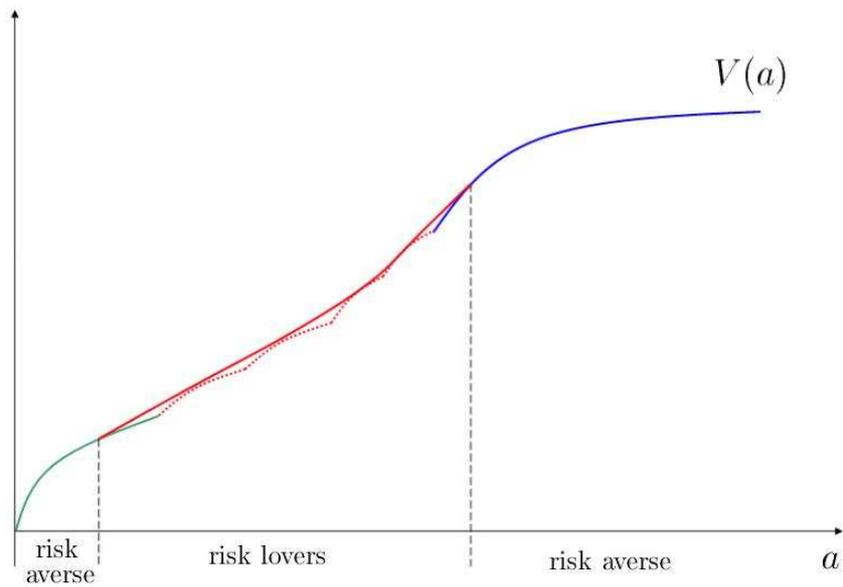


Figure 4: The shape of the indirect utility function for $\beta(1+r) < 1$ in a continuous time model.

interval (a_∞^*, a_T^*) as it is shown in Figure 4. However, this result should still be established rigorously (either by rewriting the model in continuous time or by using some limiting argument).

Figure 5 and 6 illustrate how the shape of $V(a)$ changes as $\beta(1+r)$ increases. Since Proposition 1 holds for any value of $\beta(1+r)$, relationship (11) implies that $V'(a_0) = V'(a_1) = \dots = V'(a_T)$ if $\beta(1+r) = 1$ and $V'(a_0) > V'(a_1) > \dots > V'(a_T)$ if $\beta(1+r) > 1$. At the same time, the relationship $a_0 < a_1 < \dots < a_T$ holds independently of the value of $\beta(1+r)$ (of course, the actual values of a_0, a_1, \dots, a_T as well as T are different for different $\beta(1+r)$). This suggests that as the time period shrinks, those consumer who save in order to switch to higher housing consumption become risk-neutral when $\beta(1+r) = 1$ and risk-averse when $\beta(1+r) > 1$. Note that even in the latter case the fact that $V(a)$ is concave within (a_∞^*, a_T^*) does not necessarily imply that these consumers are more risk averse than what they would be if there were no consumption commitments.

In short, we have illustrated that if in the presence of consumption commitments some consumers decide to postpone switching to a higher level commitment good, these consumers' risk aversion is not necessarily magnified by the presence of consumption commitments. We have shown that risk preferences of such consumers are determined by the value of $\beta(1+r)$: it turns out that for the values of $\beta(1+r)$ consistent with the typical predictions of the general equilibrium macroeconomic models ($\beta(1+r) = 1$ in complete market economies and $\beta(1+r) < 1$ in the models with uninsured risk), the consumers of this type become risk-neutral or even risk-lovers because they have committed to low housing value in the beginning of their life.

Notice that the risk attitudes of the consumers inside the interval (a_∞^*, a_T^*) (risk averse/risk loving/risk neutral) are solely determined by the value of $\beta(1+r)$ and do not depend on the curvature of the instantaneous utility function $u(\cdot, h)$ or of the value function $V(a)$. To better understand the intuition behind this finding, we first construct two examples suggesting why $\beta(1+r)$ plays such an important role and then explain why the lessons learnt from these examples could be applied to the general version of our model.

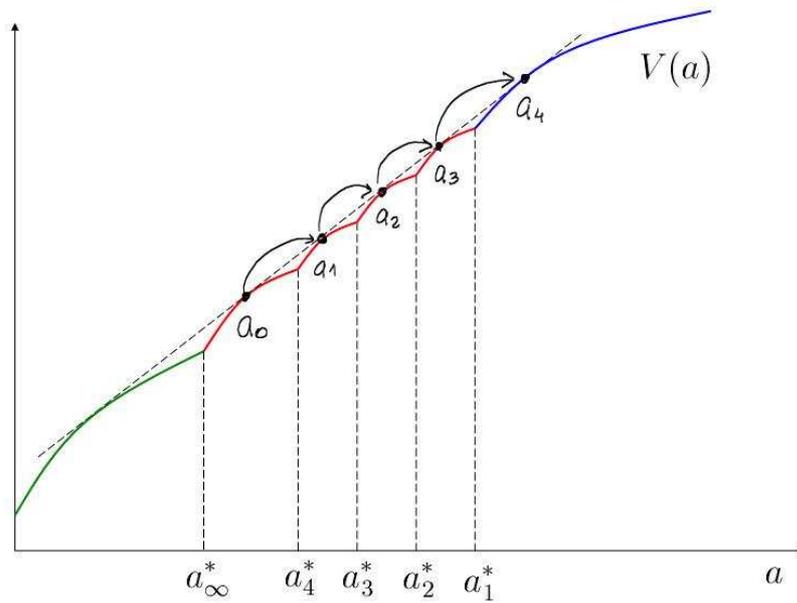


Figure 5: The effect of consumption commitments on the agents' indirect utility, $\beta(1+r) = 1$.

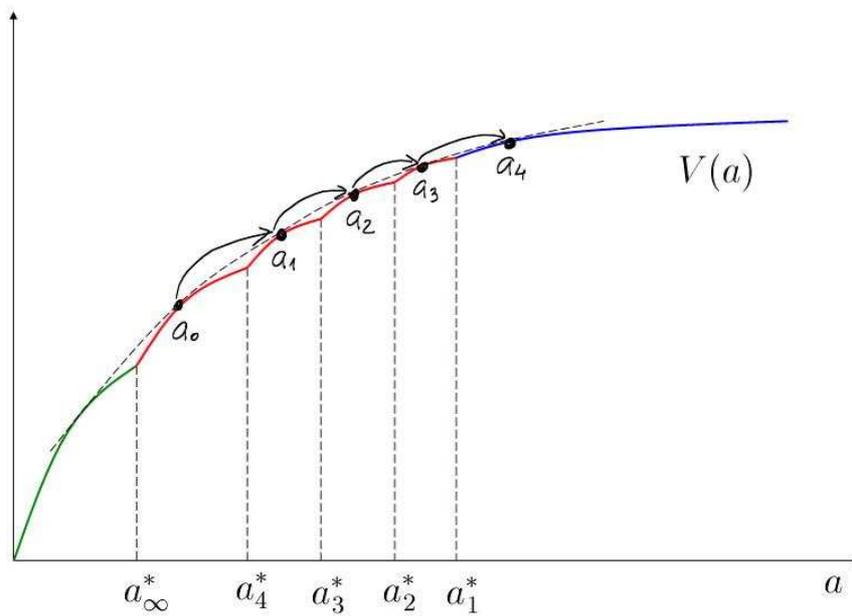


Figure 6: The effect of consumption commitments on the agents' indirect utility, $\beta(1+r) > 1$.

Example 1: Switching time and risk preferences

Consider the agent who saves $s > 0$ in period $t = 0$ and by the time when the total cumulative return on his savings becomes equal to Y , the agent can take an action which would raise his life-time utility by Δ (measured in utils). Suppose that in period 0 the agent is allowed to either save s in a risk-free bond or to take a fair lottery and then save its outcome in a risk-free bond. If the lottery is successful, the agent would be able to experience utility jump Δ sooner; if the outcome of the lottery is not successful, the utility jump would occur later. In order to figure out whether the agent would be willing to take a lottery first, let us compute the present value $PV(s)$ of the utility gain associated with the future utility jump Δ if the agent saves s in a risk-free bond.

For brevity, let us assume for this part only that the time is continuous. The agent would accumulate amount Y by the time period x such that $s \exp(rx) = Y$. The present value of the utility jump then would be equal to $PV(s) = \exp(-\rho x)\Delta$ (where $\beta = \frac{1}{1+\rho}$), which could be rearranged as

$$PV(s) = \exp(-\rho x)\Delta = (\exp(-rx))^{\rho/r}\Delta = (s/Y)^{\rho/r}\Delta.$$

Notice that $PV(s)$ is strictly concave in s if $\rho < r$ ($\beta(1+r) > 1$), strictly convex if $\rho > r$ ($\beta(1+r) < 1$) and is linear if $\rho = r$ ($\beta(1+r) = 1$). In our benchmark model exactly the same conditions on $\beta(1+r)$ determine whether the agent is risk averse, risk lover or risk neutral while he is saving in order to move to a new house.

Example 2: Separable $u(c, h)$, no borrowing constraints

Now we illustrate that the previous example can be mimicked by a special case of our benchmark model. Suppose that the utility function is separable in consumption and housing, $u(c, h) = v_c(c) + \mu v_h(h)$, and that there are no borrowing constraints. Then the decision problem of the agent can be rewritten as:

$$\begin{aligned} \max_{\{c_t\}, T} & \sum_{t=0}^{+\infty} \beta^t v_c(c_t) + \frac{v_h(h^1)}{1-\beta} + \beta^T \frac{v_h(h^2) - v_h(h^1)}{1-\beta} \\ \text{s.t.} & \frac{1+r}{r}y + a_0 = \sum_{t=0}^{+\infty} \frac{c_t}{(1+r)^t} + \frac{1+r}{r}h^1 + \frac{1}{(1+r)^T} \left[\eta + \frac{1+r}{r}(h^2 - h^1) \right] \end{aligned} \tag{12}$$

The similarities between decision problem (12) and Example 1 studied above become obvious once we realize that the consumer's behavior can be interpreted in the following way. The agent borrows against his future income in period 0 and opens two risk-free bank accounts: the savings on the first bank account will be used to finance the stream of consumption expenses $\{c_t\}$ and housing payments of size h^1 throughout his life; while the savings on the second bank account (of the initial size $s = \frac{1}{(1+r)^T}[\eta + \frac{1+r}{r}(h^2 - h^1)]$) are used to finance the switch from h^1 to h^2 in period T . At the time of the switch the agent's life-time utility would jump up by $\Delta = \frac{v_h(h^2) - v_h(h^1)}{1-\beta}$. Thus, according to our conclusions in Example 1, the agent should strictly prefer to invest his savings in the second bank account in a safe asset if $\beta(1+r) > 1$ and would be risk loving if $\beta(1+r) < 1$.

General intuition

These two examples suggest that the risk attitudes of the consumers inside the intermediate interval (a_∞^*, a_T^*) are explained by the fact that the savings decision of the consumer who eventually plans to switch to a new house could be separated into two parts. More specifically, such agent makes savings for two different purposes: (i) to smooth the marginal utility of his flexible consumption $u_1(c, h_t)$ (which is the same as the marginal value $V'(a)$) over time and (ii) to raise his life-time utility level at the moment when the switch happens. The second type of savings governs the agent's risk preferences during the path of wealth accumulation towards the switch; that is why all the findings from Example 1 also apply in a general model.

Contribution to the literature on dynamic models with Friedman-Savage instantaneous utility

Our findings suggest that in an infinite horizon model in which $\beta(1+r) \geq 1$ the consumers can completely eliminate the non-concavity of their value function (driven by the discreteness of their choice set) if they can choose optimally their savings plan as well the switching moment from one option to another. This observation contributes to the literature on dynamic models with Friedman-Savage utility function. Friedman and Savage (1948) has initially proposed to study non-concave instantaneous utility function as an explanation for why consumers simultaneously buy insurance and lottery tickets. In a later paper, Bailey et. al. (1980) have argued that in a dynamic environment the consumers might be able to eliminate

non-concavity in instantaneous utility function if they have access to borrowing and lending technology (therefore suggesting that Friedman-Savage utility would not necessarily explain simultaneous demand for lotteries and insurance in a dynamic world).

However, a recent work by Hartley and Farrel (2002) had pointed out that Bailey et. al.'s argument works only if the consumer's income is drawn from the set of countable points and the demand for lotteries may pertain otherwise. Their model, similar to ours, was set up in discrete time. The authors derived the conditions on the relationship between $\beta(1+r)$ and the consumers' income level under which the consumers would remain in the proximity to the small kink points (appearing due to time discreteness) during their transition path, and, thereby, would eventually be willing to take a small lottery in order to eliminate such 'local non-concavity'. However, the authors have not mentioned that as time period shrinks to zero, such local non-concavity should not be an issue any more, and, correspondingly, they did not discuss that the incentives to take lotteries would completely disappear if $\beta(1+r) \geq 1$.

2.4 Incentives to Postpone Switching

As a partial case, it might happen that $V(a) = \max\{V_\infty(a), V_0(a)\}$, i.e. the intermediate interval within which the consumer decides to remain in an old house for a while and switch to a new house later is empty. The previous literature analyzing the effects of consumption commitments has studied the models in which this particular case would be the only solution to the agents' decision problem, either because the static models were analyzed (Postlewaite et. al. 2006) or because attention was restricted to the dynamic models in which there was no motive for postponing the moment of adjusting the level of commitment good (Chetty and Szeidl 2007). Figure 7 illustrates why this solution suggests that consumption commitments are likely to magnify risk aversion. If the adjustment cost were such that $\lim_{h' \rightarrow h} \eta^n(h, h') = 0$ (superindex n stands for 'no commitment'), the consumers would adjust their housing consumption continuously in response to the changes in the wealth level. Under some assumptions on $\eta^n(h, h')$, the consumers' indirect life-time utility $V^n(a)$ would be strictly concave. To make the cases with and without commitment comparable, let us assume that h^1 is the optimal choice of housing at some $a^1 > 0$, h^2 is the

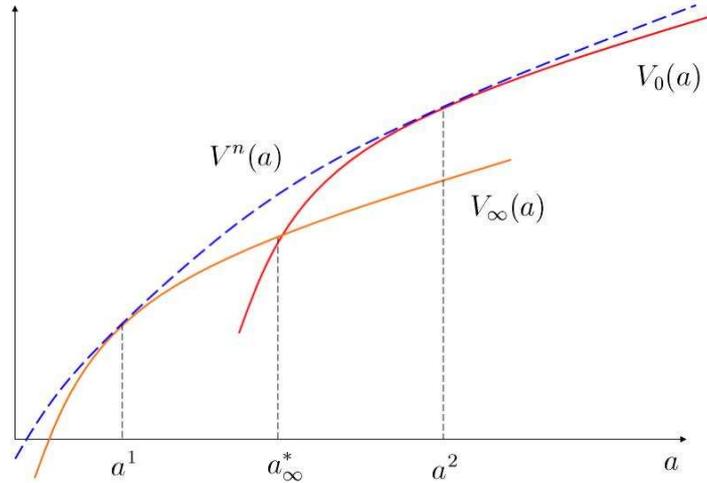


Figure 7: The effect of consumption commitments on the agents' indirect utility when $V(a) = \max\{V_\infty(a), V_0(a)\}$. $V^n(a)$ is the consumers' indirect utility in the absence of consumption commitments ($\eta^n(h, h') \rightarrow 0$ as $h' \rightarrow h$); a^1 is the optimal choice of housing at a^1 , a^2 is the optimal choice of housing at a^2 and $\eta^n(h, h')$ is such that $\eta^n(h^1, h^2) = \eta^1$.

optimal choice of housing at $a^2 > 0$ and $\eta^n(h, h')$ is such that $\eta^n(h^1, h^2) = \eta^1$. Then $V^n(a)$ is tangent to $V(a) = \max\{V_\infty(a), V_0(a)\}$ at $a = a^1$ and $a = a^2$ as it is shown in Figure 7.

Since consumer's attitudes towards risk are characterized by the curvature of the value functions $V(a)$ and $V^n(a)$, it is obvious that housing commitments increase risk aversion of the consumers with $a_0 = a^1$ and $a_0 = a^2$. Under additional assumptions on $\eta^n(h, h')$ and the relative curvature of $u(c)$ and $v(h)$, it can be shown that the curvature of $V(a)$ exceeds the curvature of $V^n(a)$ for all $a \in (\hat{a}_\infty, a_\infty^*)$ and all $a \in (\hat{a}_\infty, +\infty)$, thus implying that housing commitments magnify risk aversion independently of the initial wealth level. Intuitively, in the presence of housing commitments small changes in initial wealth should be absorbed only by changes in food consumption, which might generate substantial welfare losses if $u(c)$ has high risk aversion coefficient. Thus the existing theoretical studies came to the conclusion that consumption commitments make consumers more risk averse.⁵

⁵Some papers, e.g. Chetty and Szeidl 2006, compare the model with commitments to the environment without adjustment costs; they also conclude that commitments magnify risk aversion, even though the result cannot be illustrated graphically using Figure 7 – the agent's value in the absence of adjustment costs should exceed $V(a)$ for all $a \neq a^1$.

Our analysis in the previous section suggested that consumption commitments can have a very different effect on consumers' risk attitudes if the prospects of adjusting the level of commitment good in the future alters consumers' savings behavior. At the same time, Proposition 1 did not say how wide the intermediate interval might be or, in other words, how likely it is that the consumers' wealth falls into it. The following proposition addresses this question.

PROPOSITION 2

Suppose that $V_0(a)$ and $V_\infty(a)$ satisfy the conditions of Proposition 1. Then

- (i) $V(a) > V_\infty(a)$ for all $a \geq \underline{a}$ when $\beta(1+r) > 1$;*
- (ii) either $V(a) > V_\infty(a)$ for all $a \geq \underline{a}$ or $V(a)$ is the concave envelope of $\max\{V_\infty(a), V_0(a)\}$ when $\beta(1+r) = 1$.*

In other words, we should expect the the intermediate wealth intervals $(\hat{a}_\infty, a_\infty^*)$ might be sizeable. In our model the consumer might decide to postpone switching into a new house for four different reasons:

- (a) due to discreteness of available housing levels (in order to average out the life-time housing expenditures if the consumer's per period income is such that he would prefer to choose some housing level $h^* \in (h^1, h^2)$ at the moment of switching);
- (b) in order to decrease the present value of the switching cost η if $r > 0$ (thereby raising the present value of the life-time wealth);
- (c) in order to accumulate sufficient funds necessary for switching (to pay for η and higher housing expenditures) if the borrowing constraints are present and the consumer does not have much wealth in the beginning of his life;
- (d) if $\beta(1+r) > 0$, the consumer's optimal consumption and wealth profiles are increasing over time, thus even the consumers with low initial wealth levels would eventually want to switch to a bigger house.

All these four reasons creating incentives to postpone switching in our model are absent in the dynamic model developed in Chetty and Szeidl (2007) to analyze the effects of consumption commitments on risk preferences. The authors study the

environment in which the consumers can choose the level of housing flexibly at the moment of switching. They also assume that the interest rate r is equal to zero, the time discount rate β is equal to one (thus they have a finite horizon model) and the consumers can borrow against their future income. That is why in Chetty and Szeidl's environment all the consumers either choose to remain in the old house forever or decide to switch to a new house right away.

In the rest of the paper we generalize our benchmark model by allowing for a more flexible choice of housing at the moment of switching. With this modification, our model becomes a generalization of Chetty and Szeidl (2007), which allows us to analyze whether either of the last three reasons (b)-(d) in the above list might be responsible for generating a sizable intermediate interval, in which consumption commitments do not necessarily magnify consumers' risk aversion.

3 Extension: Flexible Choice of Housing

Suppose that the agent can choose any house of the size $h \geq h^1$ if he decides to adjust his housing consumption. Suppose also that the housing size can be adjusted only once, i.e. the value of the agent can be easily found if he had already moved to a new house. Then the decision problem of the agent who still remains in the old house h^1 can be written as

$$V(a) = \max_{h^* \geq h^1} \{V(a; h^*)\}, \quad (13)$$

where $V(a; h^*)$ is the value of the agent choosing whether to stay in h^1 or to move into a house of the given size h^* . Obviously, $V(a; h^*)$ is the same as the value function $V(a)$ characterized in the previous Section (with $h^2 = h^*$). Thus for any $h^* > h^1$ the agents planning to switch to h^* become risk lovers if $\beta(1+r) < 1$, risk neutral if $\beta(1+r) = 1$ or risk averse if $\beta(1+r) > 1$.

In this environment the agent might decide to postpone switching to a new house for three reasons: to decrease the present value of the adjustment cost (if $r > 0$), to accumulate the necessary funds if the borrowing constraints are binding, and to support an increasing consumption profile if $\beta(1+r) > 0$. The last case is of the least interest to us because the agent is risk averse throughout the whole life-time (though not necessarily more risk-averse than what he would be in the model without fixed

adjustment costs) and also because such relationship between interest rate and the time discount factor is unlikely to arise in an equilibrium of a macroeconomic model. Thus in what follows we will focus primarily on the first two cases and analyze the role of the positive interest rate and the borrowing constraints in shaping consumers' risk preferences in the presence of fixed adjustment costs.

3.1 Numerical Exercise

The dynamic model of Chetty and Szeidl (2007) predicts that the agents make all housing decisions in period 0 because the following assumptions were imposed in their environment: (a) $r = 0$, $\beta(1 + r) = 1$ and (b) consumers can borrow against their future income. As it was suggested in the previous Section, relaxing any of these two assumptions might lead to the appearance of the intermediate interval $(\hat{a}_\infty, a_\infty^*)$, within which consumption commitments do not necessarily magnify risk aversion. The goal of the following numerical exercise is to analyze how wide intermediate intervals might be generated if we 'generalize' Chetty and Szeidl (2007) by relaxing either of the two above assumptions.

Suppose that the utility function is separable in consumption and housing,

$$u(c, h) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + \mu \frac{h^{1-\sigma_h}}{1-\sigma_h}$$

and assume that $\sigma_c = 4$ and $\sigma_h = 1$. Let us normalize per period income to $y = 1$ and choose μ in such a way that in the model without fixed adjustment costs the agent with zero initial wealth spends 50% of his life-time income on housing ($\mu = 8$). Correspondingly, the initial housing commitment is chosen to be $h^1 = 0.5$. Finally, we set the adjustment cost η equal to 10% of the initial housing commitment.⁶

3.1.1 The role of $r > 0$

Suppose that there are no borrowing constraints. For the benchmark case we assume that $\beta(1 + r) = 1$ but, in contrast to Chetty and Szeidl (2007), we set $\beta = 0.96$ and $r = 0.0417$, which implies that $\eta = 1.25$.

⁶These parameters are chosen similar to the ones used in the benchmark simulation of Chetty and Szeidl (2007), see column 2 of Table 2.

Table 1: The size of the intermediate interval with $r > 0$ and without borrowing constraints

$r(\%)$	$\beta(1+r)$	(\hat{a}_∞, a_1^*)	$(\Delta_{min}, \Delta_{max})$
4.17	1.000	(1.58,8.74)	(0.06,0.34)
3.90	0.997	(2.27,7.88)	(0.08,0.29)
3.70	0.995	(2.80,6.83)	(0.09,0.24)
3.50	0.994	(3.22,6.31)	(0.10,0.20)
3.30	0.992	(3.60,5.18)	(0.12,0.17)
3.10	0.989	(3.84,4.63)	(0.13,0.14)
2.90	0.988	empty	empty

For these parameter values the agents with initial wealth between 1.58 and 8.74 would start saving in order to eventually move into a bigger house (see the first row in Table 1). This intermediate interval shrinks as the interest rate falls (keeping the rest of the parameters constant) and it becomes empty when reaches $r = 0.029$.

The last column in Table 1 provides a possible measure of how wide the computed intermediate intervals are. This is done in the following way. Suppose that the the initial housing commitment h^1 is endogenously chosen in period 0 at no cost. Denote by a_0^* the initial wealth at which $h^1 = 0.5$ is optimal for the given parameter values ($a_0^* = 0$ for our case). Obviously, since for all cases reported in Table 1, $\beta(1+r) \leq 1$, the agent who initially chose h^1 optimally would not be willing to move into a bigger house later in his life. However, if such agent had experienced an unexpected permanent income shock in period 0, his life-time wealth would rise and he might decide to adjust his housing consumption even though it is costly (since there are no borrowing constraints in this environment, such an income shock is isomorphic to a change in the initial wealth level). The last column in Table 1 reports the range of percentage deviations in income that would induce the agent to start saving for a bigger house. For instance, if the agent's annual income rises by more than 6% but less than 34%, the agent adjusts his savings plan in order to eventually move into a new house and becomes temporarily risk neutral (since $\beta(1+r) = 1$). Income shocks of this size are not uncommon in the data

Table 2: The size of the intermediate interval
for different values of η , $\beta(1+r) = 1$

η	% of $\frac{1+r}{r}h^1$	(\hat{a}_∞, a_1^*)	$(\Delta_{min}, \Delta_{max})$
1.25	10	(1.58, 8.74)	(0.06, 0.34)
1.0	8.0	(1.47, 7.94)	(0.05, 0.31)
0.8	6.4	(1.37, 7.12)	(0.05, 0.28)
0.6	4.8	(1.23, 6.26)	(0.04, 0.24)
0.4	3.2	(1.06, 5.40)	(0.04, 0.21)
0.2	1.6	(0.85, 3.98)	(0.02, 0.15)
0	0	empty	empty

(e.g. Kydland 1984); most numerical macroeconomic models with incomplete markets assume that the standard deviation of the annual income is around 20% (e.d. Hugget 1996).

Table 2 illustrates the interval (\hat{a}_∞, a_1^*) remains sizeable even for relatively small values of the switching cost η (we keep $r = 0.0417$ for different values of η). As η decreases, $V_0(a)$ shifts to the left and $V_\infty(a)$ remains unchanged. Since for $\beta(1+r)$ the interval wealth threshold levels \hat{a}_∞ and a_1^* are the tangent points with the common tangent line to $V_\infty(a)$ and $V_0(a)$ respectively, the interval (\hat{a}_∞, a_1^*) remains wide as η decreases.

3.1.2 The role of the borrowing constraints

In the model with the borrowing constraints, the consumers with relatively low wealth might be not able to adjust their housing consumption either because it is not feasible (i.e., if $a + y \leq \eta$) or because paying the cost η upfront would require a significant decrease in food consumption for some time.

Numerical results in Table 3 illustrate that the borrowing constraints can indeed cause consumers to postpone switching to a new housing level; while in the models with exactly the same parameter values and no borrowing constraints all the consumers would either choose to remain in house h^1 forever or switch to a new house right away.

Table 3: The effects of the borrowing constraints
on the risk preferences of the consumers, $y = 1.1$

r	$\beta(1+r)$	(\hat{a}_∞, a_1^*)	T	RP for 10% lot. with $a \geq 0$	RP for 10% lot. no b.c.
2.9%	0.988	[0.55,1.67)	19	-1.0%	0.6%
2.7%	0.986	[0.62,1.58)	17	-1.2%	0.4%
2.5%	0.984	[0.67,1.51)	14	-1.4%	0.2%

As we have illustrated earlier, in the model without borrowing constraint the incentives to postpone housing adjustment disappear if the interest r falls below 3.0%. In particular, if we increase per period income to $y = 1.1$, the agents with zero initial wealth would choose to move into a new house right away. Table 1 reports that if the borrowing constraint $a \geq 0$ is imposed, then the same consumers would not be able to borrow against their future income and would choose to remain in the house h^1 forever. However, if the consumer's initial wealth falls into the interval $(0.55, 1.67)$ (which corresponds to 50% to 150% of the annual income), he would start accumulating wealth in order to eventually move into a bigger house. During this time, the consumer is risk lover (since $\beta(1+r) < 1$) and would be willing to pay the price of 1% if he were able to gamble 10% of his wealth in a fair lottery. If the borrowing constraints were not present, this consumer would take the same wealth lottery only if it offered a premium of 0.6%.

Finally, it is not surprising that outside of the interval (\hat{a}_∞, a_1^*) , the agents are more risk averse in the model with borrowing constraints than without them (not reported in Table 3). This is consistent with the standard predictions of a neoclassical growth model, in which imposing borrowing limits increases the cost of uninsured risk (e.g. Aiyagari 1994). That is why the analysis in this paper draws attention to a situation in which borrowing constraints might have a seemingly counterintuitive effect by lowering the risk aversion of consumers (in conjunction with fixed adjustment costs or discrete choice).

4 Final Remarks

In this paper we have argued that in a dynamic environment consumption commitments do not necessarily magnify risk aversion. In particular, if a consumer decides to postpone moving into a bigger house for some time, he becomes risk lover if $\beta(1+r) < 1$ or risk neutral if $\beta(1+r) = 1$. We also argue that such behavior can be driven by either of the two factors, positive interest rate (creating incentives to postpone the adjustment cost to the future) and borrowing constraints (implying that the housing adjustment might happen only if the agent accumulates sufficient amount of wealth). Previous studies of consumption commitments in dynamic setup assumed that the interest rate equals to 0 and that the consumers can easily borrow against their future income, that is why the effect of consumption commitments on risk preferences discussed in this paper did not arise in those models.

From the methodological perspective, the paper develops a simple approach to characterizing deterministic dynamic discrete choice models in discrete time. Our finding could be applied to any framework, in which the discreteness of the choice is either exogenous or arises endogenously due to the presence of fixed adjustment costs, as long as the value of switching to a new option satisfies the conditions of Proposition 1 and, prior to switching to this option, the agent's decision problem is deterministic. The example of the two closely related models to the environment studied in this paper are the models with costly technology adoption and the models of occupational choice.

Our results also contribute to the literature on dynamic models with Friedman-Savage utility function. A number of papers in the field have discussed to what extent the non-concavity in instantaneous utility function can be eliminated by the possibility of borrowing and saving. However, we are not aware of any research work that would arrive at the conclusion that the value function obtained in a dynamic setup would become concave if $\beta(1+r) \geq 1$. Even though our framework cannot be directly interpreted as a dynamic model with Friedman-Savage utility, intuition suggests that the results of Proposition 1 could also be applied in that environment.

Our analysis has two major limitations. First, the proofs heavily rely on the fact that along the transition path towards the new option the agent's decision problem is deterministic. Intuition suggests that our results should extend if we add just a bit of uncertainty or introduce very persistent income shocks. However, a rigorous

analysis of the case with uncertainty is needed.

Second, throughout the paper we assumed that the agent can adjust the level of housing only once. This was done in order to easily characterize the value of the consumer after the adjustment is made (in particular, we need to know that it is concave). However, it can be argued that this assumption is not crucial in the environment without uncertainty. If $\beta(1+r) = 1$ then the consumer would not want to adjust the level of housing more than once. If $\beta(1+r) > 1$, the consumer would want to move to a bigger house later on; and we already know that under such condition on $\beta(1+r)$ the consumer's value function becomes concave. Lastly, if $\beta(1+r) < 1$, the consumer might be willing to move into a smaller house in the future. While he is planning to do so, his wealth and consumption will be falling and thus his value function will remain concave. This suggests that our results should remain true if we allow for multiple switches; however, a more rigorous analysis is also needed to verify whether this guess is correct.

Finally, we do not endogenize the initial housing commitment h^1 (even though in the numerical example in the last section we discuss how this can be done). The type of consumers' behavior that we are interested in arises when the consumers' initial housing choice ends up being suboptimal in the long run. This outcome could be explained either by the presence of uncertainty or by the presence of the borrowing constraints. In the former case, it might happen that *ex ante* housing choice becomes suboptimal *ex post*, after the realization of the income shock. In the latter case, adding differences in the adjustment costs would do the job. For instance, if the cost of moving into a small house is smaller than the cost of moving into a big house, the agent might choose a small house in the beginning of his life (when he has little wealth) and move into a big house later on. Either of these features would have to be modelled carefully if one wants to carefully quantify the effects of consumption commitments on agents' risk preferences (i.e. in order to analyze whether consumption commitments help to resolve the private equity premium puzzle).

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6 Appendix

Proof of Lemma 1

We start by proving the second statement of the Lemma. Since $V_0(a)$ and $V_1(a)$ are strictly concave, the policy functions $a'_0(a)$ and $a'_1(a)$ are strictly increasing. Thus it is sufficient to verify that $a'_0(\hat{a}_1^*) > a_1^*$ and $a'_1(\hat{a}_1^*) < a_1^*$. We do it by contradiction. Suppose that $a'_0(\hat{a}_1^*) \leq a_1^*$. Then, by definition of a_1^* , $V_1(a'_0(\hat{a}_1^*)) \geq V_1(a_0'(\hat{a}_1^*))$. At the same time, $a'_0(\hat{a}_1^*)$ is a feasible saving rule for the consumer maximizing $TV_1(a)$. Thus

$$\begin{aligned} TV_1(\hat{a}_1^*) &> u(\hat{a}_1^* + y - h^1 - \frac{a'_0(\hat{a}_1^*)}{1+r}, h^1) + \beta V_1(a'_0(\hat{a}_1^*)) \geq \\ &\geq u(\hat{a}_1^* + y - h^1 - \frac{a'_0(\hat{a}_1^*)}{1+r}, h^1) + \beta V_0(a'_0(\hat{a}_1^*)) = TV_0(a_{t+1}^*), \end{aligned}$$

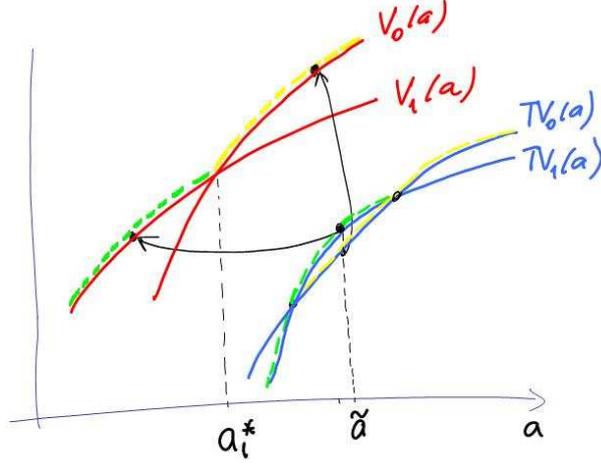


Figure 8: Illustration to the Proof of Lemma 1

which contradicts to the definition of \hat{a}_1^* . The first inequality in the above expression is strict because $TV_1(a)$ is flatter than $TV_0(a)$ at \hat{a}_1^* , so the saving decision $a'_0(\hat{a}_1^*)$ is strictly suboptimal. The inequality $a'_1(\hat{a}_1^*) < a_1^*$ can be easily verified in a similar way.

To prove (ii) we use the above property of the policy functions. Suppose that $TV_0(a)$ and $TV_1(a)$ have multiple intersections. Then there exist \tilde{a} such that $TV_0'(\tilde{a}) = TV_1'(\tilde{a})$ and $TV_1(\tilde{a}) > TV_0(\tilde{a})$ (it is easy to verify that $TV_1(\tilde{a}) < TV_0(\tilde{a})$ for sufficiently large a). Since \tilde{a} is in between the two intersection points, it follows that $a'_1(\tilde{a}) < a_1^*$ and $a'_0(\tilde{a}) > a_1^*$ and thus $TV_1(a'_1(\tilde{a})) < TV_0(a'_0(\tilde{a}))$. At the same time, $TV_0'(\tilde{a}) = TV_1'(\tilde{a})$ implies that the agents maximizing $TV_0(\tilde{a})$ and $TV_1(\tilde{a})$ have the same current utility, which contradicts to $TV_1(a'_1(\tilde{a})) < TV_0(a'_0(\tilde{a}))$. Thus $TV_1(a)$ and $TV_0(a)$ cannot have multiple intersections. ■

Proof of Proposition 1

Let us denote by a_t^* the wealth level at which $V_t(a)$ and $V_{t-1}(a)$ intersect; and by a_∞^t the wealth level at which $V_t(a)$ and $V_\infty(a)$ intersect (by (i) of Lemma 1 each pair has at most one intersection). To prove Proposition 1 it is sufficient to verify that $a_{t+1}^* < a_t^*$ as long as $V_{t+1}(a) > \max\{V_\infty(a), V_0(a), V_1(a), \dots, V_t(a)\}$ for some $a \geq \underline{a}$. This is done in two steps.

Step 1: Let's show that if $V_t(a) < \max\{V_\infty(a), V_{t-1}(a)\}$ for all $a \geq \underline{a}$ then $V_{t+1}(a) < \max\{V_\infty(a), V_t(a)\}$ for all $a \geq \underline{a}$.

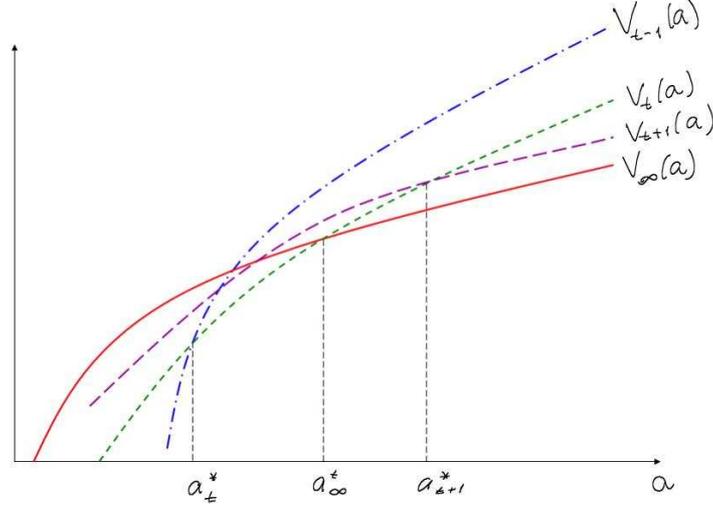


Figure 9: Illustration to Step 1 of the proof of Proposition 1

Suppose that the opposite is true and there exist $t \geq 1$ such that $V_t(a) < \max\{V_{\infty}(a), V_{t-1}(a)\}$ for all $a \geq \hat{a}_t$ but $V_{t+1}(a) \geq \max\{V_{\infty}(a), V_t(a)\}$ for some a . This implies that $a_{t+1}^* \geq a_{\infty}^* > a_t^*$ (see Figure 9). Correspondingly, $V_{t+1}(a_{t+1}^*) > V_{\infty}(a_{t+1}^*)$.

First, notice that $a'_{t+1}(a_{t+1}^*) > a_{t+1}^*$. If the opposite is true then $V_{t+1}(a_{t+1}^*) \geq V_t(a_{t+1}^*)$ and thus $V_{t+2}(a'_{t+1}(a_{t+1}^*)) \geq V_{t+1}(a'_{t+1}(a_{t+1}^*))$. Since $V_{t+2}(a)$ and $V_{t+1}(a)$ have unique intersection and $V_{t+1}(a) > V_{t+2}(a)$ for sufficiently large a , $V_{t+2}(a) \geq V_{t+1}(a) \geq V_t(a)$ for all $a \leq a_{t+1}^*$. By induction, it follows that $V_{t+k}(a) \geq V_{t+1}(a)$ for all $a \leq a_{t+1}^*$ and $k \geq 2$, which leads to contradiction because $\lim_{k \rightarrow +\infty} V_{t+k}(a) = V_{\infty}(a)$ and $V_{t+1}(a_{t+1}^*) > V_{\infty}(a_{t+1}^*)$.

Second, $a'_{t+1}(a_{t+1}^*) > a_{t+1}^*$ implies that

$$\begin{aligned} V_{t+1}(a_{t+1}^*) &= u(a_{t+1}^* + y - h^1 - \frac{a'_{t+1}(a_{t+1}^*)}{1+r}, h^1) + \beta V_t(a'_{t+1}(a_{t+1}^*)) < \\ &< u(a_{t+1}^* + y - h^1 - \frac{a'_{t+1}(a_{t+1}^*)}{1+r}, h^1) + \beta V_{t-1}(a'_{t+1}(a_{t+1}^*)) \leq \\ &\leq u(a_{t+1}^* + y - h^1 - \frac{a'_t(a_{t+1}^*)}{1+r}, h^1) + \beta V_{t-1}(a'_t(a_{t+1}^*)) = V_t(a_{t+1}^*), \end{aligned}$$

which contradicts to the definition of a_{t+1}^* ($V_{t+1}(a_{t+1}^*) = V_t(a_{t+1}^*)$).

Correspondingly, if for some T $V_{T+1}(a) < \max\{V_{\infty}(a), V_0(a), V_1(a), \dots, V_T(a)\}$ for all $a \geq \underline{a}$ then for any $k \geq 1$ $V_{T+k}(a) < \max\{V_{\infty}(a), V_0(a), V_1(a), \dots, V_T(a)\}$ for all

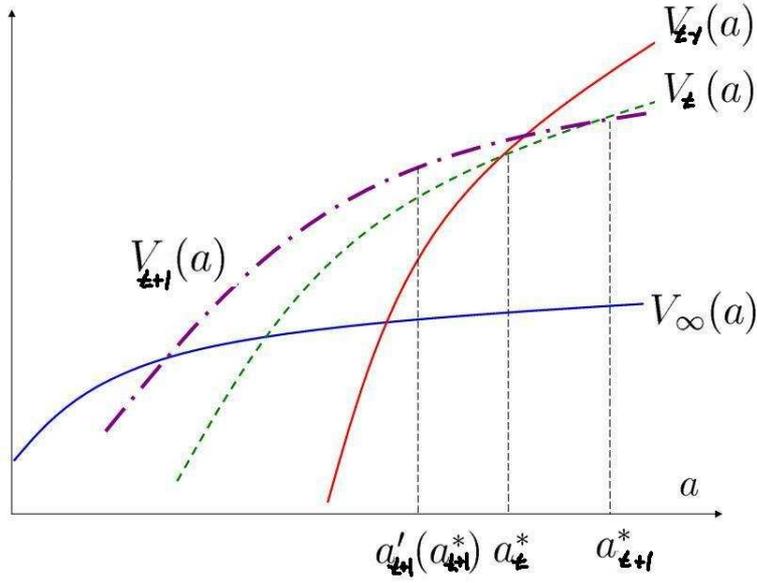


Figure 10: Illustration to Step 2 of the proof of Proposition 1

$a \geq \underline{a}$ and $V(a) = \max\{V_\infty(a), V_0(a), V_1(a), \dots, V_T(a)\}$.

Step 2: Let's verify that $a^*_{t+1} \leq a^*_t$ for all $t \leq T - 1$.

Suppose that the opposite is true and $a^*_{t+1} > a^*_t$ for some $t \leq T - 1$ (see Figure 10). Then, by (ii) of Lemma 1, $a'_{t+1}(a^*_{t+1}) < a^*_t$. Correspondingly, $V_{t+1}(a'_{t+1}(a^*_{t+1})) > V_t(a'_{t+1}(a^*_{t+1}))$. Therefore,

$$\begin{aligned}
 V_{t+2}(a^*_{t+1}) &\geq u(a^*_{t+1} + y - h^1 - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^1) + \beta V_t(a'_{t+1}(a^*_{t+1})) > \\
 &> u(a^*_{t+1} + y - h^1 - \frac{a'_{t+1}(a^*_{t+1})}{1+r}, h^1) + \beta V_{t-1}(a'_{t+1}(a^*_{t+1})) = \\
 &= V_t(a^*_{t+1}) = V_{t+1}(a^*_{t+1}).
 \end{aligned}$$

Since $V_{t+2}(a)$ and $V_{t+1}(a)$ have at most one intersection, $V_{t+2}(a) > V_{t+1}(a)$ for all $a \leq a^*_{t+1}$. Thus we can inductively apply the above argument and conclude that for all $k \geq 2$ $V_{t+k}(a) > V_{t+1}(a) > V_\infty(a)$ for all $a \leq a^*_{t+1}$, which obviously contradicts to $\lim_{k \rightarrow +\infty} V_{t+k}(a) = V_\infty(a)$. ■

Proof of Lemma 2

First, let's recall how the functions $V_0(a)$ and $V_1(a)$ are determined:

$$\begin{aligned} V_0(a) &= \max_{a' > \underline{a}} \left\{ u(a + y - h^2 - \frac{a'}{1+r}, h^2) + \beta V^2(a') \right\} = \\ &= V^2(a - \eta) \end{aligned} \quad (14)$$

$$V_1(a) = \max_{a' > \underline{a}} \left\{ u(a + y - h^1 - \frac{a'}{1+r}, h^1) + \beta V_0(a') \right\} \quad (15)$$

Denote by $c_0(a)$ and $c_1(a)$ the corresponding optimal levels of non-commitment consumption. By the envelope condition, $V_0'(a) < V_1'(a)$ implies that $u_1(c_0(a), h^2) < u_1(c_1, h^2)$. The first order conditions for the decision problems (14) and (15) implies that $u_1(c_0(a), h^2) \geq \beta(1+r)V^{2'}(a'_0(a))$ and $u_1(c_1(a), h^1) \geq \beta(1+r)V_0'(a'_1(a))$, with strict inequality if $a'_i(a) = \underline{a}$.

Notice that the decision problem (15) cannot have a corner solution at a point where $V_0'(a) < V_1'(a)$. This is because $a'_1(a) = \underline{a}$ in conjunction with $h^2 > h^1$, $\eta > 0$ and $a'_2(a) > \underline{a}$ implies that $c_0(a) < c_1(a)$, which contradicts to $u_1(c_0(a), h^2) < u_1(c_1, h^2)$. Thus the first order condition to (15) holds with equality:

$$\begin{aligned} V_0'(a) &= u_1(c_0(a), h^2) \geq \beta(1+r)V^{2'}(a_0(a)) \\ V_1'(a) &= u_1(c_1(a), h^1) = \beta(1+r)V_0'(a_1(a)) \end{aligned}$$

Thus $V_0'(a) < V_1'(a)$ implies that $u(c_0(a), h^2) > u(c_1(a), h^1)$ (since $u_{12}(a) > 0$ and $V^{2'}(a_0(a)) < V_0'(a_1(a))$). Finally, since $V_0(a) = V^2(a - \eta)$, the latter inequality implies that $V^2(a_0(a)) > V_0(a_1(a))$, which completes the proof of Lemma. ■

Proof of Proposition 2

(i) If $\beta(1+r) > 1$ then the optimal savings policy of the agent who never moves satisfies $a'_\infty(a) > a$. Correspondingly, if $V_\infty(a_\infty^t) = V_t(a_\infty^t)$ and $V'_\infty(a_\infty^t) \leq V'_t(a_\infty^t)$ for some $t \geq 1$ then $V_\infty(a_\infty^t) < V_{t+1}(a_\infty^t)$. Thus $\max\{V_0(a), V_1(a), \dots\} > V_\infty(a)$.

(ii) When $\beta(1+r) = 1$, $a'_\infty(a) = a$ and the combination of $V_\infty(a_\infty^t) = V_t(a_\infty^t)$ and $V'_\infty(a_\infty^t) < V'_t(a_\infty^t)$ for some $t \geq 1$ implies that $V_\infty(a_\infty^t) < V_{t+1}(a_\infty^t)$. Thus in the limit one of the two cases might happen: (a) $\lim_{t \rightarrow +\infty} a_\infty^t = \underline{a}$ and $\max\{V_0(a), V_1(a), \dots\} > V_\infty(a)$ or (b) $\lim_{t \rightarrow +\infty} a_\infty^t > \underline{a}$ and $\lim_{t \rightarrow +\infty} V'_t(a_\infty^t) = V'_\infty(a_\infty^t)$. ■